

Number, Relations and Functions 10 Curriculum

Implemented September 2011

Acknowledgements

The Department of Education of New Brunswick gratefully acknowledges the contributions of the following groups and individuals toward the development of the *New Brunswick Number, Relations and Functions 10 Curriculum Guide*:

- The Western and Northern Canadian Protocol (WNCP) for Collaboration in Education: *The Common Curriculum Framework for Grade 10-12 Mathematics*, January 2008. Reproduced (and/or adapted) by permission. All rights reserved.
- Newfoundland and Labrador Department of Education, Prince Edward Island Department of Education and Early Childhood Development.
- The NB High School Mathematics Curriculum Development Advisory Committee of Bev Amos, Roddie Dugay, Suzanne Gaskin, Nicole Giberson, Karen Glynn, Beverlee Gonzales, Ron Manuel, Jane Pearson, Elaine Sherrard, Alyssa Sankey (UNB), Mahin Salmani (UNB), , Maureen Tingley (UN),
- The NB Grade 10 Curriculum Development Writing Team of Heather Chamberlain, Audrey Cook, Lori-Ann Lauridsen, Tammy McIntyre, Tina Paige-Acker, Parise Plourde, Janice Shaw, Glen Spurrell, David Taylor, and Shawna Woods-Roy.
- Martha McClure, Learning Specialist, 9-12 Mathematics and Science, NB Department of Education
- The Mathematics Learning Specialists, Numeracy Leads, and Mathematics teachers of New Brunswick who provided invaluable input and feedback throughout the development and implementation of this document.

2012

New Brunswick Department of Education and
Early Childhood Development

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Curriculum Overview for Grades 10-12 Mathematics

BACKGROUND AND RATIONALE

Mathematics curriculum is shaped by a vision which fosters the development of mathematically literate students who can extend and apply their learning and who are effective participants in society.

It is essential the mathematics curriculum reflects current research in mathematics instruction. To achieve this goal, *The Common Curriculum Framework for Grades 10–12 Mathematics: Western and Northern Canadian Protocol* has been adopted as the basis for a revised mathematics curriculum in New Brunswick. The Common Curriculum Framework was developed by the seven ministries of education (Alberta, British Columbia, Manitoba, Northwest Territories, Nunavut, Saskatchewan and Yukon Territory) in collaboration with teachers, administrators, parents, business representatives, post-secondary educators and others.

The framework identifies beliefs about mathematics, general and specific student outcomes, and achievement indicators agreed upon by the seven jurisdictions. This document is based on both national and international research by the WNCP and the NCTM.

There is an emphasis in the New Brunswick curriculum on particular key concepts at each grade which will result in greater depth of understanding and ultimately stronger student achievement. There is also a greater emphasis on number sense and operations concepts in the early grades to ensure students develop a solid foundation in numeracy.

The intent of this document is to clearly communicate high expectations for students in mathematics education to all education partners. Because of the emphasis placed on key concepts at each grade level, time needs to be taken to ensure mastery of these concepts. Students must learn mathematics with understanding, actively building new knowledge from experience and prior knowledge (NCTM Principles and Standards, 2000).

BELIEFS ABOUT STUDENTS AND MATHEMATICS LEARNING

The New Brunswick Mathematics Curriculum is based upon several key assumptions or beliefs about mathematics learning which have grown out of research and practice.

These beliefs include:

- mathematics learning is an active and constructive process;
- learners are individuals who bring a wide range of prior knowledge and experiences, and who learn via various styles and at different rates;
- learning is most likely to occur when placed in meaningful contexts and in an environment that supports exploration, risk taking, and critical thinking and that nurtures positive attitudes and sustained effort; and
- learning is most effective when standards of expectation are made clear with on-going assessment and feedback.

Students are curious, active learners with individual interests, abilities and needs. They come to classrooms with varying knowledge, life experiences and backgrounds. A key component in successfully developing numeracy is making connections to these backgrounds and aspirations.

Students construct their understanding of mathematics by developing meaning based on a variety of learning experiences. This meaning is best developed when learners encounter mathematical experiences that proceed from simple to complex and from the concrete to the abstract. The use of manipulatives, visuals and a variety of pedagogical approaches can address the diversity of learning styles and developmental stages of students. At all levels of understanding students benefit from working with a variety of materials, tools and contexts when constructing meaning about new mathematical ideas. Meaningful student discussions also provide essential links among concrete, pictorial and symbolic representations of mathematics. The learning environment should value, respect and address all students' experiences and ways of thinking, so that students are comfortable taking intellectual risks, asking questions and posing conjectures. Students need to explore mathematics through solving problems in order to continue developing personal strategies and mathematical literacy. It is important to realize that it is acceptable to solve problems in different ways and that solutions may vary depending upon how the problem is understood.

Goals for Mathematically Literate Students

The main goals of mathematics education are to prepare students to:

- use mathematics confidently to solve problems
- communicate and reason mathematically
- appreciate and value mathematics
- make connections between mathematics and its applications
- commit themselves to lifelong learning
- become mathematically literate adults, using mathematics to contribute to society.

Students who have met these goals will:

- gain understanding and appreciation of the contributions of mathematics as a science, philosophy and art
- exhibit a positive attitude toward mathematics
- engage and persevere in mathematical tasks and projects
- contribute to mathematical discussions

- take risks in performing mathematical tasks
- exhibit curiosity

In order to assist students in attaining these goals, teachers are encouraged to develop a classroom atmosphere that fosters conceptual understanding through:

- taking risks
- thinking and reflecting independently
- sharing and communicating mathematical understanding
- solving problems in individual and group projects
- pursuing greater understanding of mathematics
- appreciating the value of mathematics throughout history.

Opportunities for Success

A positive attitude has a profound effect on learning. Environments that create a sense of belonging, encourage risk taking, and provide opportunities for success help develop and maintain positive attitudes and self-confidence. Students with positive attitudes toward learning mathematics are likely to be motivated and prepared to learn, participate willingly in classroom activities, persist in challenging situations and engage in reflective practices.

Teachers, students and parents need to recognize the relationship between the affective and cognitive domains, and attempt to nurture those aspects of the affective domain that contribute to positive attitudes. To experience success, students must be taught to set achievable goals and assess themselves as they work toward these goals.

Striving toward success, and becoming autonomous and responsible learners are ongoing, reflective processes that involve revisiting the setting and assessing of personal goals.

Diverse Cultural Perspectives

Students come from a diversity of cultures, have a diversity of experiences and attend schools in a variety of settings including urban, rural and isolated communities. To address the diversity of knowledge, cultures, communication styles, skills, attitudes, experiences and learning styles of students, a variety of teaching and assessment strategies are required in the classroom. These strategies must go beyond the incidental inclusion of topics and objects unique to a particular culture.

For many First Nations students, studies have shown a more holistic worldview of the environment in which they live (Banks and Banks 1993). This means that students look for connections and learn best when mathematics is contextualized and not taught as discrete components. Traditionally in Indigenous culture, learning takes place through active participation and little emphasis is placed on the written word. Oral communication along with practical applications and experiences are important to student learning and understanding. It is important that teachers understand and respond to both verbal and non-verbal cues to optimize student learning and mathematical understandings.

Instructional strategies appropriate for a given cultural or other group may not apply to all students from that group, and may apply to students beyond that group. Teaching for diversity will support higher achievement in mathematics for all students.

Adapting to the Needs of All Learners

Teachers must adapt instruction to accommodate differences in student development as they enter school and as they progress, but they must also avoid gender and cultural biases. Ideally, every student should find his/her learning opportunities maximized in the mathematics classroom. The reality of individual student differences must not be ignored when making instructional decisions.

As well, teachers must understand and design instruction to accommodate differences in student learning styles. Different instructional modes are clearly appropriate, for example, for those students who are primarily visual learners versus those who learn best by doing. Designing classroom activities to support a variety of learning styles must also be reflected in assessment strategies.

Universal Design for Learning

The New Brunswick Department of Education and Early Childhood Development's definition of inclusion states that every child has the right to expect that his or her learning outcomes, instruction, assessment, interventions, accommodations, modifications, supports, adaptations, additional resources and learning environment will be designed to respect his or her learning style, needs and strengths.

Universal Design for Learning is a "...framework for guiding educational practice that provides flexibility in the ways information is presented, in the ways students respond or demonstrate knowledge and skills, and in the ways students are engaged." It also "...reduces barriers in instruction, provides appropriate accommodations, supports, and challenges, and maintains high achievement expectations for all students, including students with disabilities and students who are limited English proficient" (CAST, 2011).

In an effort to build on the established practice of differentiation in education, the Department of Education and Early Childhood Development supports *Universal Design for Learning* for all students. New Brunswick curricula are created with universal design for learning principles in mind. Outcomes are written so that students may access and represent their learning in a variety of ways, through a variety of modes. Three tenets of universal design inform the design of this curriculum. Teachers are encouraged to follow these principles as they plan and evaluate learning experiences for their students:

- **Multiple means of representation:** provide diverse learners options for acquiring information and knowledge
- **Multiple means of action and expression:** provide learners options for demonstrating what they know
- **Multiple means of engagement:** tap into learners' interests, offer appropriate challenges, and increase motivation

For further information on *Universal Design for Learning*, view online information at <http://www.cast.org/>.

Connections across the Curriculum

The teacher should take advantage of the various opportunities available to integrate mathematics and other subjects. This integration not only serves to show students how mathematics is used in daily life, but it helps strengthen the students' understanding of mathematical concepts and provides them with opportunities to practise mathematical skills. There are many possibilities for integrating mathematics in literacy, science, social studies, music, art, and physical education.

NATURE OF MATHEMATICS

Mathematics is one way of trying to understand, interpret and describe our world. There are a number of components that define the nature of mathematics and these are woven throughout this document. These components include: **change, constancy, number sense, relationships, patterns, spatial sense and uncertainty.**

Change

It is important for students to understand that mathematics is dynamic and not static. As a result, recognizing change is a key component in understanding and developing mathematics. Within mathematics, students encounter conditions of change and are required to search for explanations of that change. To make predictions, students need to describe and quantify their observations, look for patterns, and describe those quantities that remain fixed and those that change. For example, the sequence 4, 6, 8, 10, 12, ... can be described as:

- skip counting by 2s, starting from 4
- an arithmetic sequence, with first term 4 and a common difference of 2
- a linear function with a discrete domain

(Steen, 1990, p. 184).

Students need to learn that new concepts of mathematics as well as changes to already learned concepts arise from a need to describe and understand something new. Integers, decimals, fractions, irrational numbers and complex numbers emerge as students engage in exploring new situations that cannot be effectively described or analyzed using whole numbers.

Students best experience change to their understanding of mathematical concepts as a result of mathematical play.

Constancy

Different aspects of constancy are described by the terms stability, conservation, equilibrium, steady state and symmetry (AAAS–Benchmarks, 1993, p. 270). Many important properties in mathematics and science relate to properties that do not change when outside conditions change. Examples of constancy include:

- the area of a rectangular region is the same regardless of the methods used to determine the solution
- the sum of the interior angles of any triangle is 180°
- the theoretical probability of flipping a coin and getting heads is 0.5.

Some problems in mathematics require students to focus on properties that remain constant. The recognition of constancy enables students to solve problems involving constant rates of change, lines with constant slope, direct variation situations or the angle sums of polygons.

Many important properties in mathematics do not change when conditions change. Examples of constancy include:

- the conservation of equality in solving equations
- the sum of the interior angles of any triangle
- the theoretical probability of an event.

Some problems in mathematics require students to focus on properties that remain constant. The recognition of constancy enables students to solve problems such as those involving constant rates of change, lines with constant slope, or direct variation situations.

Number Sense

Number sense, which can be thought of as deep understanding and flexibility with numbers, is the most important foundation of numeracy (British Columbia Ministry of Education, 2000, p. 146). Continuing to foster number sense is fundamental to growth of mathematical understanding.

A true sense of number goes well beyond the skills of simply counting, memorizing facts and the situational rote use of algorithms. Students with strong number sense are able to judge the reasonableness of a solution, describe relationships between different types of numbers, compare quantities and work with different representations of the same number to develop a deeper conceptual understanding of mathematics.

Number sense develops when students connect numbers to real-life experiences and when students use benchmarks and referents. This results in students who are computationally fluent and flexible with numbers and who have intuition about numbers. Evolving number sense typically comes as a by-product of learning rather than through direct instruction. However, number sense can be developed by providing mathematically rich tasks that allow students to make connections.

Patterns

Mathematics is about recognizing, describing and working with numerical and non-numerical patterns. Patterns exist in all of the mathematical topics, and it is through the study of patterns that students can make strong connections between concepts in the same and different topics.

Working with patterns also enables students to make connections beyond mathematics. The ability to analyze patterns contributes to how students understand their environment. Patterns may be represented in concrete, visual, auditory or symbolic form. Students should develop fluency in moving from one representation to another.

Students need to learn to recognize, extend, create and apply mathematical patterns. This understanding of patterns allows students to make predictions and justify their

reasoning when solving problems. Learning to work with patterns helps develop students' algebraic thinking, which is foundational for working with more abstract mathematics.

Relationships

Mathematics is used to describe and explain relationships. Within the study of mathematics, students look for relationships among numbers, sets, shapes, objects, variables and concepts. The search for possible relationships involves collecting and analyzing data, analyzing patterns and describing possible relationships visually, symbolically, orally or in written form.

Spatial Sense

Spatial sense involves the representation and manipulation of 3-D objects and 2-D shapes. It enables students to reason and interpret among 3-D and 2-D representations.

Spatial sense is developed through a variety of experiences with visual and concrete models. It offers a way to interpret and reflect on the physical environment and its 3-D or 2-D representations.

Some problems involve attaching numerals and appropriate units (measurement) to dimensions of objects. Spatial sense allows students to make predictions about the results of changing these dimensions.

Spatial sense is also critical in students' understanding of the relationship between the equations and graphs of functions and, ultimately, in understanding how both equations and graphs can be used to represent physical situations

Uncertainty

In mathematics, interpretations of data and the predictions made from data may lack certainty.

Events and experiments generate statistical data that can be used to make predictions. It is important to recognize that these predictions (interpolations and extrapolations) are based upon patterns that have a degree of uncertainty. The quality of the interpretation is directly related to the quality of the data. An awareness of uncertainty allows students to assess the reliability of data and data interpretation.

Chance addresses the predictability of the occurrence of an outcome. As students develop their understanding of probability, the language of mathematics becomes more specific and describes the degree of uncertainty more accurately. This language must be used effectively and correctly to convey valuable messages.

ASSESSMENT

Ongoing, interactive assessment (*formative assessment*) is essential to effective teaching and learning. Research has shown that formative assessment practices produce significant and often substantial learning gains, close achievement gaps and build students' ability to learn new skills (Black & William, 1998, OECD, 2006). Student involvement in assessment promotes learning. Interactive assessment, and encouraging self-assessment, allows students to reflect on and articulate their understanding of mathematical concepts and ideas.

Assessment in the classroom includes:

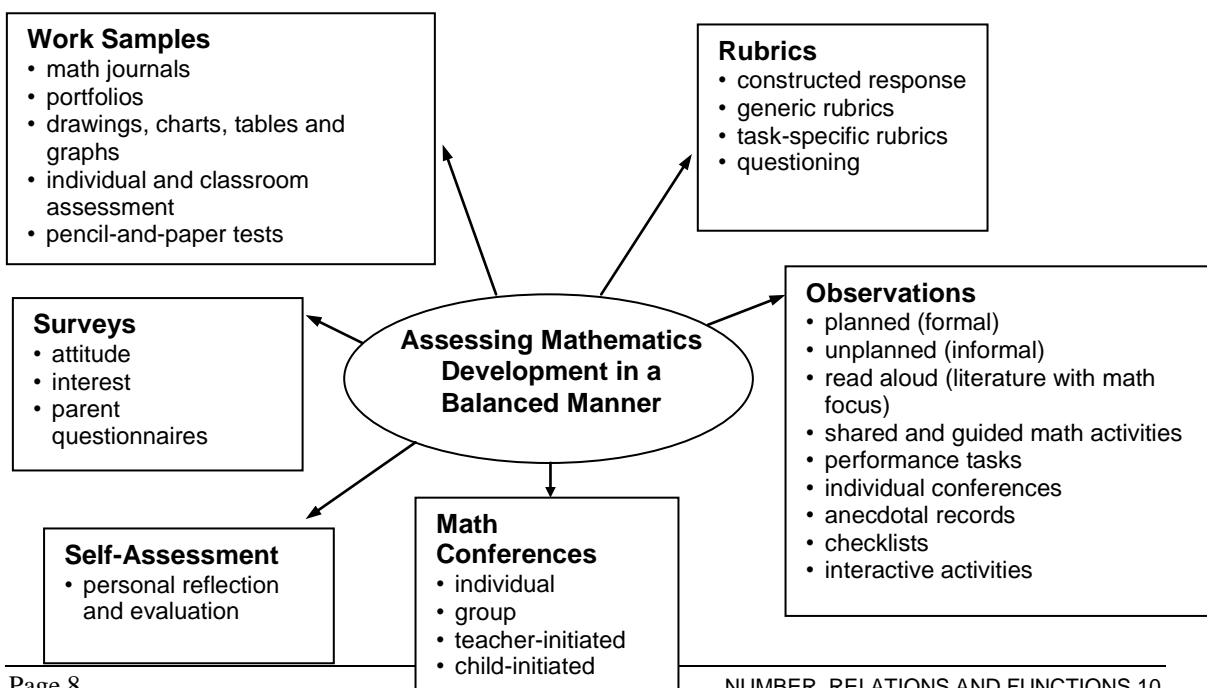
- providing clear goals, targets and learning outcomes
- using exemplars, rubrics and models to help clarify outcomes and identify important features of the work
- monitoring progress towards outcomes and providing feedback as necessary
- encouraging self-assessment
- fostering a classroom environment where conversations about learning take place, where students can check their thinking and performance and develop a deeper understanding of their learning (Davies, 2000)

Formative assessment practices act as the scaffolding for learning which, only then, can be measured through summative assessment. *Summative assessment*, or assessment of learning, tracks student progress, informs instructional programming and aids in decision making. Both forms of assessment are necessary to guide teaching, stimulate learning and produce achievement gains.

Student assessment should:

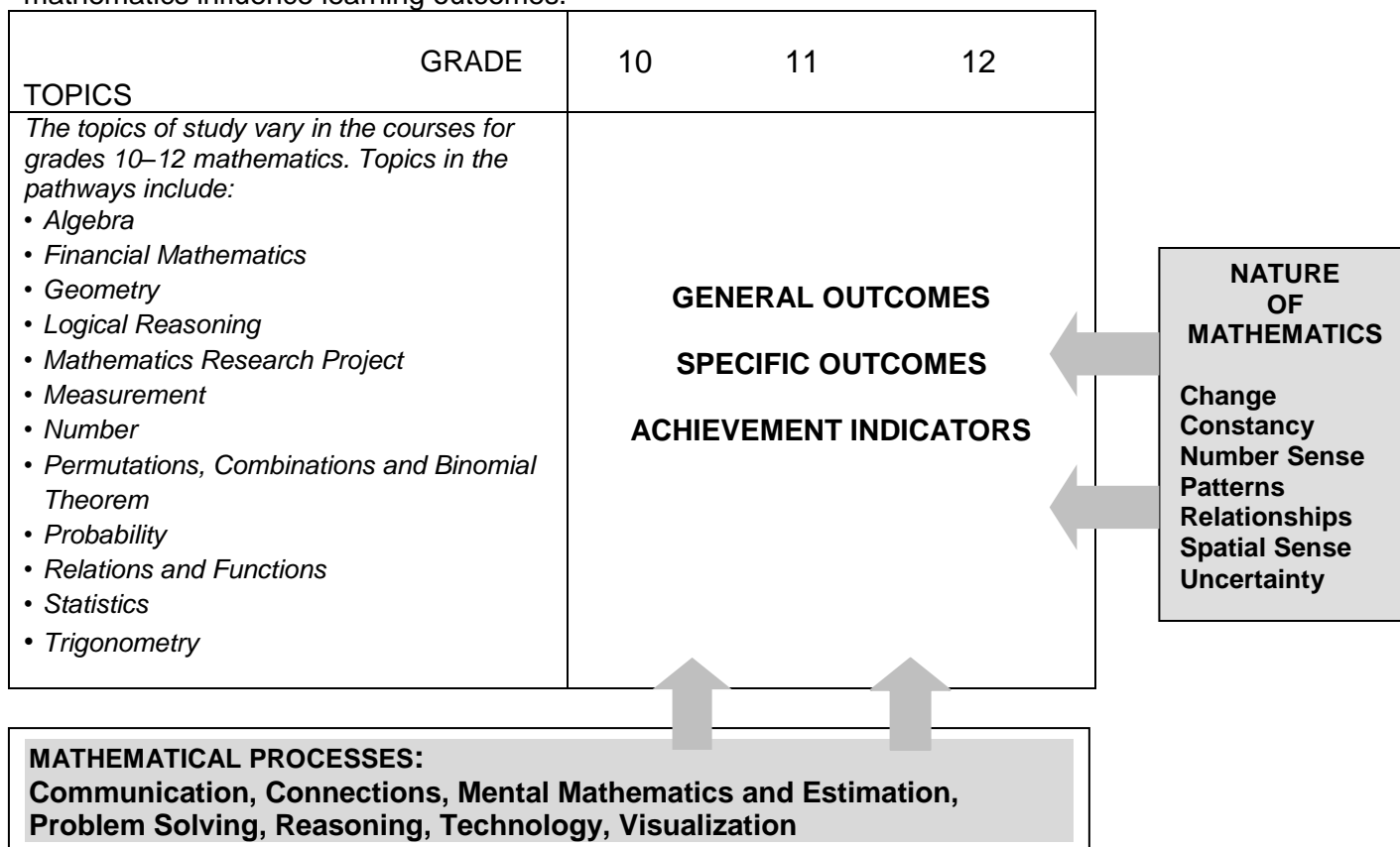
- align with curriculum outcomes
- use clear and helpful criteria
- promote student involvement in learning mathematics during and after the assessment experience
- use a wide variety of assessment strategies and tools
- yield useful information to inform instruction

(adapted from: NCTM, *Mathematics Assessment: A practical handbook*, 2001, p.22)



CONCEPTUAL FRAMEWORK FOR 10-12 MATHEMATICS

The chart below provides an overview of how mathematical processes and the nature of mathematics influence learning outcomes.



MATHEMATICAL PROCESSES

There are critical components that students must encounter in a mathematics program in order to achieve the goals of mathematics education and encourage lifelong learning in mathematics.

Students are expected to:

- communicate in order to learn and express their understanding of mathematics (Communications: C)
- connect mathematical ideas to other concepts in mathematics, to everyday experiences and to other disciplines (Connections: CN)
- demonstrate fluency with mental mathematics and estimation (Mental Mathematics and Estimation: ME)
- develop and apply new mathematical knowledge through problem solving (Problem Solving: PS)
- develop mathematical reasoning (Reasoning: R)
- select and use technologies as tools for learning and solving problems (Technology: T)
- develop visualization skills to assist in processing information, making connections and solving problems (Visualization: V).

The New Brunswick Curriculum incorporates these seven interrelated mathematical processes that are intended to permeate teaching and learning.

Communication [C]

Students need opportunities to read about, represent, view, write about, listen to and discuss mathematical ideas. These opportunities allow students to create links between their own language and ideas, and the formal language and symbols of mathematics.

Communication is important in clarifying, reinforcing and modifying ideas, knowledge, attitudes and beliefs about mathematics. Students should be encouraged to use a variety of forms of communication while learning mathematics. Students also need to communicate their learning using mathematical terminology.

Communication can help students make connections among concrete, pictorial, symbolic, verbal, written and mental representations of mathematical ideas.

Emerging technologies enable students to engage in communication beyond the traditional classroom to gather data and share mathematical ideas.

Problem Solving [PS]

Problem solving is one of the key processes and foundations within the field of mathematics. Learning through problem solving should be the focus of mathematics at all grade levels. Students develop a true understanding of mathematical concepts and procedures when they solve problems in meaningful contexts. Problem solving is to be employed throughout all of mathematics and should be embedded throughout all the topics.

When students encounter new situations and respond to questions of the type, *How would you...?* or *How could you ...?*, the problem-solving approach is being modelled. Students develop their own problem-solving strategies by listening to, discussing and trying different strategies.

In order for an activity to be problem-solving based, it must ask students to determine a way to get from what is known to what is sought. If students have already been given ways to solve the problem, it is not a problem, but practice. Students should not know the answer immediately. A true problem requires students to use prior learnings in new ways and contexts. Problem solving requires and builds depth of conceptual understanding and student engagement. Students will be engaged if the problems relate to their lives, cultures, interests, families or current events.

Both conceptual understanding and student engagement are fundamental in moulding students' willingness to persevere in future problem-solving tasks. Problems are not just simple computations embedded in a story, nor are they contrived. They are tasks that are rich and open-ended, so there may be more than one way of arriving at a solution or there may be multiple answers. Good problems should allow for every student in the class to demonstrate their knowledge, skill or understanding. Problem solving can vary from being an individual activity to a class (or beyond) undertaking.

In a mathematics class, there are two distinct types of problem solving: solving contextual problems outside of mathematics and solving mathematical problems. Finding the maximum profit given manufacturing constraints is an example of a contextual problem, while seeking and developing a general formula to solve a quadratic equation is an example of a mathematical problem.

Problem solving can also be considered in terms of engaging students in both inductive and deductive reasoning strategies. As students make sense of the problem, they will be creating conjectures and looking for patterns that they may be able to generalize. This part of the problem-solving process often involves inductive reasoning. As students use approaches to solving the problem they often move into mathematical reasoning that is deductive in nature. It is crucial that students be encouraged to engage in both types of reasoning and be given the opportunity to consider the approaches and strategies used by others in solving similar problems.

Problem solving is a powerful teaching tool that fosters multiple, creative and innovative solutions. Creating an environment where students openly look for, and engage in, finding a variety of strategies for solving problems empowers students to explore alternatives and develops confident, cognitive mathematical risk-takers.

Connections [CN]

Contextualization and making connections to the experiences of learners are powerful processes in developing mathematical understanding. When mathematical ideas are connected to each other or to real-world phenomena, students can begin to view mathematics as useful, relevant and integrated.

Learning mathematics within contexts and making connections relevant to learners can validate past experiences, and increase student willingness to participate and be actively engaged. The brain is constantly looking for and making connections.

“Because the learner is constantly searching for connections on many levels, educators need to orchestrate the experiences from which learners extract understanding... Brain research establishes and confirms that multiple complex and concrete experiences are essential for meaningful learning and teaching” (Caine and Caine, 1991, p. 5).

Mental Mathematics and Estimation [ME]

Mental mathematics is a combination of cognitive strategies that enhance flexible thinking and number sense. It is calculating mentally without the use of external memory aids.

Mental mathematics enables students to determine answers without paper and pencil. It improves computational fluency by developing efficiency, accuracy and flexibility.

“Even more important than performing computational procedures or using calculators is the greater facility that students need—more than ever before—with estimation and mental mathematics” (National Council of Teachers of Mathematics, May 2005).

Students proficient with mental mathematics *“become liberated from calculator dependence, build confidence in doing mathematics, become more flexible thinkers and are more able to use multiple approaches to problem solving”* (Rubenstein, 2001).

Mental mathematics *“provides a cornerstone for all estimation processes offering a variety of alternate algorithms and non-standard techniques for finding answers”* (Hope, 1988).

Estimation is a strategy for determining approximate values or quantities, usually by referring to benchmarks or using referents, or for determining the reasonableness of calculated values. Students need to know how, when and what strategy to use when estimating. Estimation is used to make mathematical judgments and develop useful, efficient strategies for dealing with situations in daily life. When estimating, students need to learn which strategy to use and how to use it.

Technology [T]

Technology can be used effectively to contribute to and support the learning of a wide range of mathematical outcomes. Technology enables students to explore and create patterns, examine relationships, test conjectures and solve problems.

Calculators and computers can be used to:

- explore and demonstrate mathematical relationships and patterns
- organize and display data
- generate and test inductive conjectures
- extrapolate and interpolate
- assist with calculation procedures as part of solving problems
- increase the focus on conceptual understanding by decreasing the time spent on repetitive procedures
- reinforce the learning of basic facts
- develop personal procedures for mathematical operations
- model situations
- develop number and spatial sense.

Technology contributes to a learning environment in which the curiosity of students can lead to rich mathematical discoveries at all grade levels. The use of technology should not replace mathematical understanding. Instead, technology should be used as one of a variety of approaches and tools for creating mathematical understanding.

Visualization [V]

Visualization “involves thinking in pictures and images, and the ability to perceive, transform and recreate different aspects of the visual-spatial world” (Armstrong, 1993, p. 10). The use of visualization in the study of mathematics provides students with opportunities to understand mathematical concepts and make connections among them.

Visual images and visual reasoning are important components of number, spatial and measurement sense. Number visualization occurs when students create mental representations of numbers. Being able to create, interpret and describe a visual representation is part of spatial sense and spatial reasoning. Spatial visualization and spatial reasoning enable students to describe the relationships among and between 3-D objects and 2-D shapes.

Measurement visualization goes beyond the acquisition of specific measurement skills. Measurement sense includes the ability to determine when to measure and when to estimate and involves knowledge of several estimation strategies (Shaw and Cliatt, 1989, p. 150).

Visualization is fostered through the use of concrete materials, technology and a variety of visual representations. It is through visualization that abstract concepts can be understood concretely by the student. Visualization is a foundation to the development of abstract understanding, confidence and fluency.

Reasoning [R]

Mathematical reasoning helps students think logically and make sense of mathematics. Students need to develop confidence in their abilities to reason and justify their mathematical thinking.

Questions that challenge students to think, analyze and synthesize help them develop an understanding of mathematics. All students need to be challenged to answer questions such as, *Why do you believe that's true/correct?* or *What would happen if*

Mathematical experiences provide opportunities for students to engage in inductive and deductive reasoning. Students use inductive reasoning when they explore and record results, analyze observations, make generalizations from patterns and test these generalizations. Students use deductive reasoning when they reach new conclusions based upon the application of what is already known or assumed to be true. The thinking skills developed by focusing on reasoning can be used in daily life in a wide variety of contexts and disciplines.

ESSENTIAL GRADUATION LEARNINGS

Graduates from the public schools of Atlantic Canada will be able to demonstrate knowledge, skills, and attitudes in the following essential graduation learnings. These learnings are supported through the outcomes described in this curriculum document.

Aesthetic Expression

Graduates will be able to respond with critical awareness to various forms of the arts and be able to express themselves through the arts.

Citizenship

Graduates will be able to assess social, cultural, economic, and environmental interdependence in a local and global context.

Communication

Graduates will be able to use the listening, viewing, speaking, reading and writing modes of language(s) as well as mathematical and scientific concepts and symbols to think, learn, and communicate effectively.

Personal Development

Graduates will be able to continue to learn and to pursue an active, healthy lifestyle.

Problem Solving

Graduates will be able to use the strategies and processes needed to solve a wide variety of problems, including those requiring language, mathematical, and scientific concepts.

Technological Competence

Graduates will be able to use a variety of technologies, demonstrate an understanding of technological applications, and apply appropriate technologies for solving problems

PATHWAYS AND TOPICS

The Common Curriculum Framework for Grades 10–12 Mathematics on which the New Brunswick Grades 10-12 Mathematics curriculum is based, includes pathways and topics rather than strands as in *The Common Curriculum Framework for K–9 Mathematics*. In New Brunswick all Grade 10 students share a common curriculum covered in two courses: *Geometry, Measurement and Finance 10* and *Number, Relations and Functions 10*. Starting in Grade 11, three pathways are available: *Finance and Workplace, Foundations of Mathematics*, and *Pre-Calculus*.

Each topic area requires that students develop a conceptual knowledge base and skill set that will be useful to whatever pathway they have chosen. Students are encouraged to cross pathways to follow their interests and to keep their options open. The topics covered within a pathway are meant to build upon previous knowledge and to progress from simple to more complex conceptual understandings.

Goals of Pathways

The goals of all three pathways are to provide prerequisite attitudes, knowledge, skills and understandings for specific post-secondary programs or direct entry into the work force. All three pathways provide students with mathematical understandings and critical-thinking skills. It is the choice of topics through which those understandings and skills are developed that varies among pathways. When choosing a pathway, students should consider their interests, both current and future. Students, parents and educators are encouraged to research the admission requirements for post-secondary programs of study as they vary by institution and by year.

Design of Pathways

Each pathway is designed to provide students with the mathematical understandings, rigour and critical-thinking skills that have been identified for specific post-secondary programs of study and for direct entry into the work force.

The content of each pathway has been based on the *Western and Northern Canadian Protocol (WNCP) Consultation with Post-Secondary Institutions, Business and Industry Regarding Their Requirements for High School Mathematics: Final Report on Findings* and on consultations with mathematics teachers.

Financial and Workplace Mathematics

This pathway is designed to provide students with the mathematical understandings and critical-thinking skills identified for entry into some college programs and for direct entry into the work force. Topics include financial mathematics, algebra, geometry, measurement, number, statistics and probability.

Foundations of Mathematics

This pathway is designed to provide students with the mathematical understandings and critical-thinking skills identified for post-secondary studies in programs that do not require the study of theoretical calculus. Topics include financial mathematics, geometry, measurement, number, logical reasoning, relations and functions, statistics and probability.

Pre-calculus

This pathway is designed to provide students with the mathematical understandings and critical-thinking skills identified for entry into post-secondary programs that require the study of theoretical calculus. Topics include algebra and number, measurement, relations and functions, trigonometry, and permutations, combinations and binomial theorem.

Outcomes and Achievement Indicators

The New Brunswick Curriculum is stated in terms of general curriculum outcomes, specific curriculum outcomes and achievement indicators.

General Curriculum Outcomes (GCO) are overarching statements about what students are expected to learn in each strand/sub-strand. The general curriculum outcome for each strand/sub-strand is the same throughout the pathway.

Specific Curriculum Outcomes (SCO) are statements that identify specific concepts and related skills underpinned by the understanding and knowledge attained by students as required for a given grade.

Achievement indicators are samples of how students may demonstrate their achievement of the goals of a specific outcome. The range of samples provided is meant to reflect the scope of the specific outcome. In the specific outcomes, the word *including* indicates that any ensuing items must be addressed to fully meet the learning outcome. The phrase *such as* indicates that the ensuing items are provided for clarification and are not requirements that must be addressed to fully meet the learning outcome. The word *and* used in an outcome indicates that both ideas must be addressed to fully meet the learning outcome, although not necessarily at the same time or in the same question.

Instructional Focus

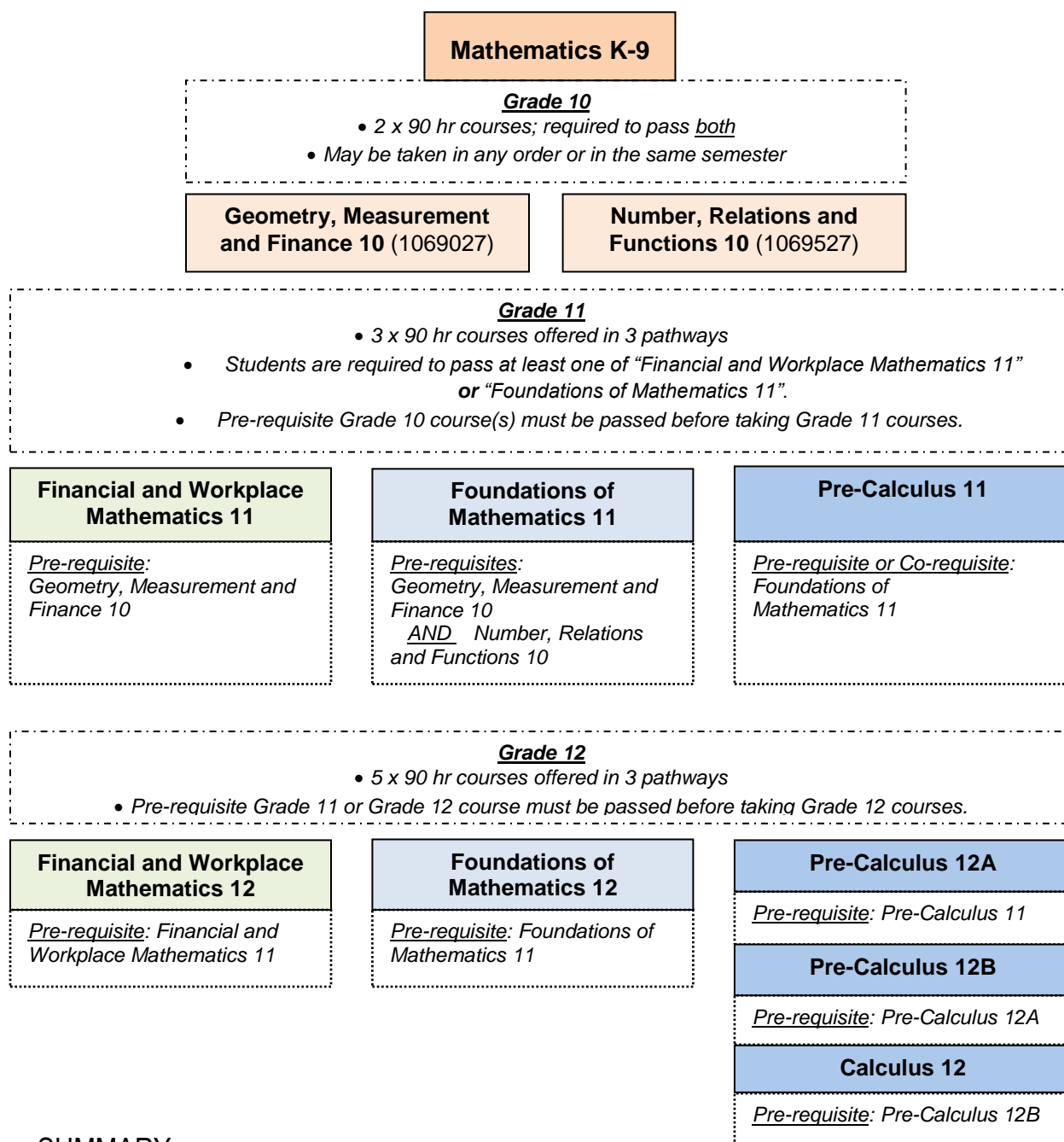
Each pathway in *The Common Curriculum Framework for Grades 10–12 Mathematics* is arranged by topics. Students should be engaged in making connections among concepts both within and across topics to make mathematical learning experiences meaningful. Teachers should consider the following points when planning for instruction and assessment.

- The mathematical processes that are identified with the outcome are intended to help teachers select effective pedagogical approaches for the teaching and learning of the outcome.
- All seven mathematical processes must be integrated throughout teaching and learning approaches, and should support the intent of the outcomes.
- Wherever possible, meaningful contexts should be used in examples, problems and projects.
- Instruction should flow from simple to complex and from concrete to abstract.
- The assessment plan for the course should be a balance of assessment for learning, assessment as learning and assessment of learning.

The focus of student learning should be on developing a conceptual and procedural understanding of mathematics. Students' conceptual understanding and procedural understanding must be directly related.

Pathways and Courses

The graphic below summarizes the pathways and courses offered.



SUMMARY

The Conceptual Framework for Grades 10–12 Mathematics describes the nature of mathematics, the mathematical processes, the pathways and topics, and the role of outcomes and achievement indicators in grades 10–12 mathematics. Activities that take place in the mathematics classroom should be based on a problem-solving approach that incorporates the mathematical processes and leads students to an understanding of the nature of mathematics.

CURRICULUM DOCUMENT FORMAT

This guide presents the mathematics curriculum by grade level so that a teacher may readily view the scope of the outcomes which students are expected to meet during that year. Teachers are encouraged, however, to examine what comes before and what follows after, to better understand how the students' learnings at a particular grade level are part of a bigger picture of concept and skill development.

The order of presentation in no way assumes or prescribes a preferred order of presentation in the classroom, but simply lays out the specific curriculum outcomes in relation to the overarching general curriculum outcomes (GCOs).

The heading of each page gives the General Curriculum Outcome (GCO), and Specific Curriculum Outcome (SCO). The key for the mathematical processes follows. A Scope and Sequence is then provided which relates the SCO to previous and next grade SCO's. For each SCO, Elaboration, Achievement Indicators, Suggested Instructional Strategies, and Suggested Activities for Instruction and Assessment are provided. For each section, the *Guiding Questions* should be considered.

GCO: General Curriculum Outcome SCO: Specific Curriculum Outcome		
Mathematical Processes		
[C] Communication Technology	[PS] Problem Solving [V] Visualization	[CN] Connections [R] Reasoning
[ME] Mental Math [T] and Estimation		
Scope and Sequence		
Previous Grade or Course SCO's	Current Grade SCO	Following Grade or Course SCO's
Elaboration		
Describes the "big ideas" to be learned and how they relate to work in previous Grades		
Guiding Questions:		
<ul style="list-style-type: none"> • <i>What do I want my students to learn?</i> • <i>What do I want my students to understand and be able to do?</i> 		
Achievement Indicators		
Describes observable indicators of whether students have met the specific outcome		
Guiding Questions:		
<ul style="list-style-type: none"> • <i>What evidence will I look for to know that learning has occurred?</i> • <i>What should students demonstrate to show their understanding of the mathematical concepts and skills?</i> 		

GCO: General Curriculum Outcome SCO: Specific Curriculum Outcome
Suggested Instructional Strategies
General approach and strategies suggested for teaching this outcome
Guiding Questions
<ul style="list-style-type: none"> • <i>What learning opportunities and experiences should I provide to promote learning of the outcomes and permit students to demonstrate their learning?</i> • <i>What teaching strategies and resources should I use?</i> • <i>How will I meet the diverse learning needs of my students?</i>
Suggested Activities for Instruction and Assessment
Some suggestions of specific activities and questions that can be used for both instruction and assessment.
Guiding Questions
<ul style="list-style-type: none"> • <i>What are the most appropriate methods and activities for assessing student learning?</i> • <i>How will I align my assessment strategies with my teaching strategies?</i>
Guiding Questions
<ul style="list-style-type: none"> • <i>What conclusions can be made from assessment information?</i> • <i>How effective have instructional approaches been?</i> • <i>What are the next steps in instruction?</i>

Number Relations and Functions 10

Specific Curriculum Outcomes

SCO AN1: Demonstrate an understanding of factors of whole numbers by determining the prime factors, greatest common factor, least common multiple, square root, and cube root.
[C, ME, R]

Algebra and Number

[C] Communication [T] Technology	[PS] Problem Solving [V] Visualization	[CN] Connections [R] Reasoning	[ME] Mental Math and Estimation
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Scope and Sequence of Outcomes

Grade Nine	Grade Ten	Grade Eleven
<p>N5 Determine the square root of positive rational numbers that are perfect squares.</p> <p>N6 Determine an approximate square root of positive rational numbers that are non-perfect squares.</p>	<p>AN1: Demonstrate an understanding of factors of whole numbers by determining the prime factors, greatest common factor, least common multiple, square root, and cube root.</p>	<p>RF2: Demonstrate an understanding of the characteristics of quadratic functions, including vertex, intercepts, domain and range, axis of symmetry. <i>(FM11)</i></p> <p>RF1: Factor polynomial expressions of the form:</p> <ul style="list-style-type: none"> • $ax^2 + bx + c, a \neq 0$ • $a^2x^2 - b^2y^2, a \neq 0, b \neq 0$ • $a(f(x))^2 + b(f(x)) + c, a \neq 0$ • $a^2(f(x))^2 - b^2(g(y))^2, a \neq 0, b \neq 0$ <p>Where a, b and c are rational numbers <i>(PC11)</i></p> <p>RF5: Solve problems that involve quadratic equations. <i>(PC11)</i></p>

ELABORATION

Students in Grade 8 investigated square roots of whole numbers up to $\sqrt{144}$, including perfect squares and estimates of non-perfect squares. They would have explored these relationships concretely, pictorially and symbolically for whole numbers.

In Grade 9, the study of square roots was extended to finding the square root of positive rational numbers that are perfect squares, including whole numbers, fractions and decimals, using benchmark perfect squares to help estimate.

In Grade 10 the focus is on factoring of whole numbers, using a variety of strategies. This is an introduction to **factoring, prime factors, greatest common factors** and **least common multiples**. Students will use a variety of strategies, including factoring, to determine the roots of perfect squares and **perfect cubes**.

To factor a whole number it is helpful to express a number as a product of its **prime factors**. A review of prime numbers, composite numbers, and prime factorization may be necessary before students are introduced to prime factors.

The numbers 0 and 1 have no prime factors. When 1 is divided by a prime number, the answer is never a whole number so 1 has no prime factors. Zero is divisible by all prime numbers ($0 \div 2 = 0, 0 \div 3 = 0$ etc.) seeming to indicate that zero has an infinite number of prime factors. However, if 2 is a factor of 0, then so is the number zero. Since division by 0 is inadmissible, 0 has no prime factors.

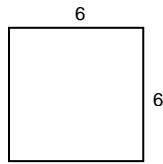
SCO AN1: Demonstrate an understanding of factors of whole numbers by determining the prime factors, greatest common factor, least common multiple, square root, and cube root.
[C, ME, R]

As an example of **prime factorization**, 24 can be expressed as a product of its prime factors: $24 = 2 \times 2 \times 2 \times 3$, or $24 = 2^3 \times 3$. To avoid confusion with the variable x , students should become familiar with use of a dot to indicate multiplication as in: $24 = 2 \cdot 2 \cdot 2 \cdot 3$.

The product of two numbers of equal value is the **square** of those numbers. If the factors are whole numbers, the product is a **perfect square**. Conversely, two factors of equal value are the **square roots** of the square. For example, 25 is the square of 5 expressed symbolically as, $5^2 = 25$ and the square root of 25 is 5 expressed symbolically as $\sqrt{25} = 5$.

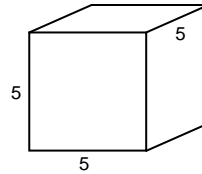
Likewise, the product of three numbers of equal value is the **cube** of those numbers. If the factors are whole numbers, the product is a **perfect cube**. Conversely, three factors of equal value are the **cube roots** of the cube. For example, 27 is the cube of 3 expressed symbolically as $3^3 = 27$, and the cube root of 27 is 3 expressed symbolically as $\sqrt[3]{27} = 3$.

A perfect square can be represented as the area of a square. The whole number side lengths are the value of the square root. A perfect cube can be represented as the volume of a cube. The whole number edge lengths are the value of the cube root. All sides are of equal lengths.



Area = 36 square units
 $\sqrt{36} = 6$

36 is a perfect square
6 is its square root



Volume = 125 cubic units
 $\sqrt[3]{125} = 5$

125 is a perfect cube
5 is its cube root

SCO AN1: Demonstrate an understanding of factors of whole numbers by determining the prime factors, greatest common factor, least common multiple, square root, and cube root.
[C, ME, R]

ACHIEVEMENT INDICATORS

- Determine the prime factors of a whole number.
- Explain why the numbers 0 and 1 have no prime factors.
- Determine, using a variety of strategies, the greatest common factor or least common multiple of a set of whole numbers, and explain the process.
- Determine, concretely, whether a given whole number is a perfect square, a perfect cube or neither.
- Determine, using a variety of strategies, the square root of a perfect square, and explain the process.
- Determine, using a variety of strategies, the cube root of a perfect cube, and explain the process.
- Solve problems that involve prime factors, greatest common factors, least common multiples, square roots or cube roots.

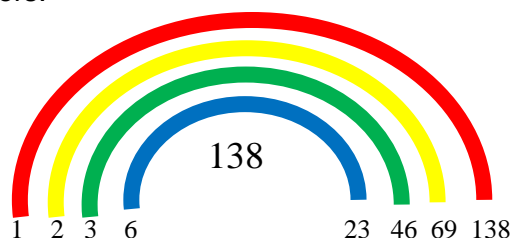
Suggested Instructional Strategies

Factoring

- Students should become familiar with a variety of ways to find the **prime factors** of a whole number including factor trees, and repeated division by prime factors.
- Students should be encouraged to use diagrams, manipulatives, factor trees and calculators, to solve problems

Greatest Common Factor

- Students should explore a variety of ways to find the **greatest common factor (GCF)** of two or more numbers. For two numbers, one way to determine the GCF is to identify the prime factors common to both numbers, and then to take the product of these factors. For example, for 60 and 24, $60 = 2 \cdot 2 \cdot 3 \cdot 5$, and $24 = 2 \cdot 2 \cdot 2 \cdot 3$. Multiplying the factors in common gives: $2 \cdot 2 \cdot 3 = 12$, so 12 is the GCF
- When introducing the greatest common factor (GCF), use division facts to determine all the factors of each number and record the factors as a rainbow, or as a list of factors.



Factors of 138

$$\begin{aligned} &1 \times 138 \\ &2 \times 69 \\ &3 \times 46 \\ &6 \times 23 \end{aligned}$$

SCO AN1: Demonstrate an understanding of factors of whole numbers by determining the prime factors, greatest common factor, least common multiple, square root, and cube root.
[C, ME, R]

Least Common Multiple

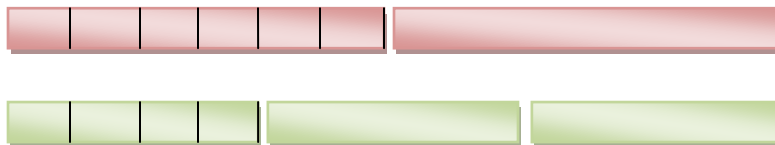
- Students should also be encouraged to explore a variety of ways to find the smallest multiple shared by two or more numbers (**least common multiple** or **LCM**). One method is to compare multiples of each number until a common multiple is found. For example, the multiples of 6 are 6, 12, 18, 24, 30, and the multiples of 10 are 10, 20, 30. The first multiple they have in common is 30, so this is the LCM.
- Once students understand the concept of LCM using models and smaller numbers, have them extend this to finding the LCM of larger numbers using other methods such as listing numbers and their multiples until the same multiple appears in all lists. For example, for the numbers 18, 20 and 30, the LCM is 180 which is the first multiple found in all 3 lists:

Multiples of 18: 18, 36, 54, 72, 90, 108, 126, 144, 162, **180**, ...

Multiples of 20: 20, 40, 60, 80, 100, 120, 140, **180**, ...

Multiples of 30: 30, 60, 90, 120, **180**, ...

- Have students use interlocking unit cubes to create multiples of two or more numbers for which they are determining the LCM. When the chains are of equal length, they will have found the LCM e.g. For 6 and 4, 2 lengths of 6 is the same length as 3 lengths of 4 so the LCM is 12. This will give students a visual image of the concept involved.



Square and Cube Roots

- For large numbers, use prime factorization to determine square roots and cube roots. For example, after determining the prime factors of the number, have students see if they can make equal groups of 2 (square roots) or of 3 (cube roots). As an extension have them do this with numbers that have 4th and 5th roots.

SCO AN1: Demonstrate an understanding of factors of whole numbers by determining the prime factors, greatest common factor, least common multiple, square root, and cube root.
[C, ME, R]

Suggested Questions (Q) and Activities (Act) for Instruction and Assessment

- Q** Determine the first 10 prime numbers and explain your strategy for finding these numbers and testing that they are prime.
- Q** Draw a factor tree for 10, 60 and 120 to determine the prime factors.
(For enrichment have students work with larger numbers and encourage them to express the prime factors in simplified form e.g. the prime factors of 3300 are $2 \cdot 2 \cdot 3 \cdot 5 \cdot 5 \cdot 11$ and could be written as $2^2 \cdot 3 \cdot 5^2 \cdot 11$.)
- Q** Explain the difference between listing the factors of a number and listing the multiples of a number.
- Q** Complete the following:
- 1) Express each number as a product:
a) 12 b) 28 c) 63
 - 2) Factor completely:
a) $4xy^2$ b) $18a^2b^3$ c) $36x^2yz^2$
 - 3) Determine the GCF of each pair of numbers:
a) 15, 20 b) 16, 24 c) 28, 42
 - 4) Determine the GCF of each pair of monomials:
a) $4a, 6a$ b) $2x^2, 3x$ c) $12abc, 3abc$
d) $9mn^2, 8mn$ e) $6x^2y^2, 9xy$
- Act** To review their current knowledge and ensure students understand the relationship between a squared number and the shape of a square, use 1cm grid paper and ask them if they can form a square that contains a given area. For example, ask students if they can draw a square on grid paper with the area of 269 cm^2 . Make sure to include area examples that are not perfect squares.
- Q** Write 729 as a product of its prime numbers. Determine whether it is a perfect square and/or a perfect cube through grouping of the prime factors.
(Note to teacher: $729 = 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3$ The prime factors can be grouped as $(3 \cdot 3 \cdot 3)(3 \cdot 3 \cdot 3)$ to give a square root of 27, or they can be grouped as $(3 \cdot 3)(3 \cdot 3)(3 \cdot 3)$ to give a cube root of 9. In this example the number 729 is both a perfect square and a perfect cube)
- Q** You have two buckets that hold 4 litres and 5 litres respectively. You need to fill two fish tanks. What is the least number of buckets you would need to fill both tanks to the same volume?

SCO AN2: Demonstrate an understanding of irrational numbers by representing, identifying, simplifying and ordering irrational numbers.
[CN, ME, R, V]

[C] Communication
[T] Technology

[PS] Problem Solving
[V] Visualization

[CN] Connections
[R] Reasoning

[ME] Mental Math
and Estimation

Scope and Sequence of Outcomes

Grade Nine	Grade Ten	Grade Eleven
<p>N3: Demonstrate an understanding of rational numbers by:</p> <ul style="list-style-type: none"> comparing and ordering rational numbers solving problems that involve arithmetic operations on rational numbers. <p>N5: Determine the square root of positive rational numbers that are perfect squares.</p> <p>N6: Determine an approximate square root of positive rational numbers that are non-perfect squares.</p>	<p>AN2: Demonstrate an understanding of irrational numbers by representing, identifying, simplifying and ordering irrational numbers.</p>	<p>RF2: Demonstrate an understanding of the characteristics of quadratic functions including vertex, intercepts, domain and range, axis of symmetry. <i>(FM11)</i></p> <p>AN2: Solve problems that involve operations on radicals and radical expressions with numerical and variable radicands. <i>(PC11)</i></p> <p>AN3: Solve problems that involve radical equations (limited to square roots). <i>(PC11)</i></p>

ELABORATION

Students had experience calculating ratios, and working with integers, decimals, and fractions in middle school. In Grade 9, students were introduced to operations with negative fractions, and the concept of rational numbers. Rational numbers are numbers that can be written as fractions, ratios, or repeating or terminating decimals.

In Grade 10 students will be introduced to the concept of **irrational** numbers defined as numbers that cannot be written as a fraction. When expressed as a decimal these numbers do not repeat or terminate. Students will determine the relationship between irrational numbers and natural, whole, integer, rational and real numbers. They will then use a variety of strategies (not including using a calculator) to estimate the values of irrational numbers to locate them on a number line.

Natural, whole and **integer** number sets are discrete and do not include fractions or decimals.

Natural numbers (1, 2, 3, 4, 5 ...)

Whole numbers (0, 1, 2, 3, 4 ...)

Integer numbers (... - 3, -2, -1, 0, 1, 2, 3 ...)

The **rational** number set includes all numbers that can be written as a fraction, which includes all numbers in the Natural, Whole and Integer sets (above), with the addition of numbers that can be written with repeating or terminating decimals.

Examples of rational numbers include:

$$\sqrt{27} = 3, \quad \frac{15}{3}, \quad \frac{-3}{1}, \quad \frac{1}{6} = 0.1\dot{6}, \quad 1\frac{1}{4} = 1.25, \quad \frac{2}{7} = 0.285714285714 \dots$$

SCO AN2: Demonstrate an understanding of irrational numbers by representing, identifying, simplifying and ordering irrational numbers.
[CN, ME, R, V]

The **irrational** number set is made up of numbers with non-repeating decimals.

Examples of irrational numbers include:

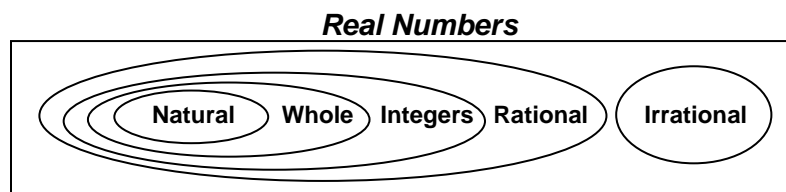
$$\pi = 3.1415926535897 \dots$$

$$\sqrt{2} = 1.41421356237 \dots$$

$$\text{the Golden Ratio} = 1.6180339887 \dots$$

$$\sqrt{99} = 9.949874371066 \dots$$

The **real** number set includes all rational numbers, and all irrational numbers.



When written as a radical, irrational numbers can be identified as those for which the radicand is not a perfect square, a perfect cube, or a perfect multiple with reference to the index e.g.

Irrational

$$\sqrt{60} = \sqrt{2 \times 2 \times 3 \times 5} = 2\sqrt{15}$$

$$\sqrt{10} = \sqrt{2 \times 5}$$

$$\sqrt[3]{9} = \sqrt[3]{3 \times 3}$$

Rational

$$\sqrt{121} = \sqrt{11 \times 11} = 11$$

$$\sqrt[3]{8} = \sqrt[3]{2 \times 2 \times 2} = 2$$

$$\sqrt[4]{81} = \sqrt[4]{3 \times 3 \times 3 \times 3} = 3$$

For the radical $\sqrt[a]{b}$, b is the **radicand**, and a is the **index**.

Entire radicals have a **numerical coefficient** of one (e.g. $\sqrt[3]{16}$, $\sqrt{200}$).

Mixed radicals have a numerical coefficient other than one (e.g. $4\sqrt{3}$, $3\sqrt[3]{2}$).

SCO AN2: Demonstrate an understanding of irrational numbers by representing, identifying, simplifying and ordering irrational numbers.
[CN, ME, R, V]

ACHIEVEMENT INDICATORS

- Sort a set of numbers into rational and irrational numbers.
- Determine an approximate value of a given irrational number.
- Approximate the locations of irrational numbers on a number line, using a variety of strategies, and explain the reasoning.
- Order a set of irrational numbers on a number line.
- Express a radical as a mixed radical in simplest form (limited to numerical radicands).
- Express a mixed radical as an entire radical (limited to numerical radicands).
- Explain, using examples, the meaning of the index of a radical.
- Represent, using a graphic organizer, the relationship among the subsets of the real numbers (natural, whole, integer, rational, irrational).

Suggested Instructional Strategies

- A review of rational numbers, and square roots (perfect and non-perfect, with a focus on estimation) is important before discussing irrational numbers.
- To test prior knowledge, have students place all types of rational numbers (fractions, integers, decimals, whole numbers, radicals with a perfect square) on a number line.
- Have students sort a mix of rational and irrational numbers into their respective categories. Encourage students to convert decimals to fractions and vice versa in order to complete this task.
- Place irrational numbers on a number line using benchmarks e.g. Students should be able to place $\sqrt{9}$ easily, and $\sqrt{10}$ can be placed close to it.
- As a check of understanding, provide a list of irrational numbers already placed on a number line with some errors peppered through it. Then ask students to find the errors and have them explain how they came to that conclusion.
- A good understanding of radicals will be indicated when students demonstrate the ability to switch back and forth between mixed radicals and entire radicals. Different methods can be used. For example:

Mixed to Entire

$$4\sqrt{3} = \sqrt{4 \times 4 \times 3} = \sqrt{48}$$

$$3\sqrt[3]{2} = \sqrt[3]{3 \cdot 3 \cdot 3 \cdot \sqrt[3]{2}} = \sqrt[3]{27 \cdot \sqrt[3]{2}} = \sqrt[3]{27 \cdot 2} = \sqrt[3]{54}$$

Entire to Mixed

$$\sqrt[3]{16} = \sqrt[3]{8 \cdot 2} = 2\sqrt[3]{2}$$

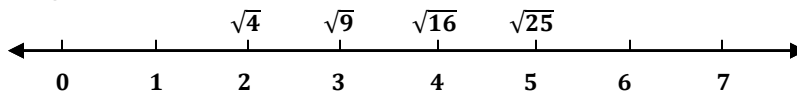
$$\sqrt{200} = \sqrt{2 \cdot 2 \cdot 2 \cdot 5 \cdot 5} = \sqrt{(2 \cdot 2) \cdot 2 \cdot (5 \cdot 5)} = 2 \cdot 5\sqrt{2} = 10\sqrt{2}$$

SCO AN2: Demonstrate an understanding of irrational numbers by representing, identifying, simplifying and ordering irrational numbers.
[CN, ME, R, V]

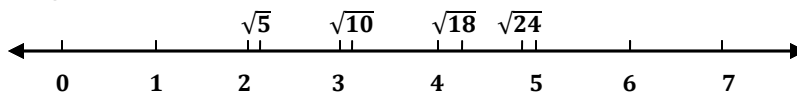
Suggested Questions (Q) and Activities (Act) for Instruction and Assessment

Act Lead students through a series of number line activities that develop their level of understanding of perfect squares, non-perfect squares, entire and mixed radicals, so students can clearly see the relation between numerical values and radical expressions.

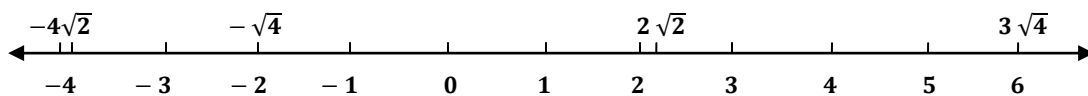
Step 1



Step 2



Step 3



Act Place a variety of rational and irrational numbers in each of 3 envelopes. Have students place these values in order on a number line. The difficulty level should increase with each successive envelope.

- 1st set: $\sqrt{25}$, -3 , -1.5 , $\frac{1}{4}$, $\sqrt{4}$, $-1\frac{1}{3}$
- 2nd set: π , $\sqrt{\frac{1}{9}}$, $-\sqrt{25}$, $1.321698345\dots$, $\frac{4}{7}$, $-\frac{3}{8}$, $\sqrt{10}$
- 3rd set: $\sqrt{18}$, $3\sqrt{2}$, $-1\frac{1}{7}$, $\sqrt[4]{81}$, $-2.876143792\dots$

Act Create a Jeopardy game that challenges students to deal with all number subsets – natural, whole, integers, rational, real, irrational.

Q Joan simplified $\sqrt{200}$ to $2\sqrt{50}$, thinking that was the simplest form. Find and correct the error.

(teacher note: Guide students to:

$$2\sqrt{25 \times 2} = 2 \times 5\sqrt{2} = 10\sqrt{2} \quad \text{OR} \quad \sqrt{100 \times 2} = \sqrt{100} \times \sqrt{2} = 10\sqrt{2})$$

SCO AN3: Demonstrate an understanding of powers with integral and rational components.
[C, CN, PS, R]

[C] Communication
[T] Technology

[PS] Problem Solving
[V] Visualization

[CN] Connections
[R] Reasoning

[ME] Mental Math
and Estimation

Scope and Sequence of Outcomes

Grade Nine	Grade Ten	Grade Eleven
<p>N1 Demonstrate an understanding of powers with integral bases (excluding base 0) and whole number exponents by:</p> <ul style="list-style-type: none"> representing repeated multiplication using powers using patterns to show that a power with an exponent of zero is equal to one solving problems involving powers. <p>N2 Demonstrate an understanding of operations on powers with integral bases (excluding base 0) and whole number exponents.</p>	<p>AN3 Demonstrate an understanding of powers with integral and rational components.</p>	<p>AN2 Solve problems that involve operations on radicals and radical expressions with numerical and variable radicands.(PC11)</p> <p>AN3 Solve problems that involve radical equations (limited to square roots).(PC11)</p>

ELABORATION

In Grade 9 students investigated the concepts of exponents, bases and powers and developed an understanding of powers with integral bases and whole number exponents.* They explored the generalized exponent laws using numerical components, emphasizing order of operations. Common misunderstandings were addressed such as: $6^5 + 6^2 \neq 6^7$, $(2^3)^2 \neq 2^5$, $(5^3 \times 5^4) \neq 5^{12}$.

It will likely be necessary to review the five exponent laws before starting this outcome. It may be helpful to expose students to strings of questions that demonstrate the pattern behind the rule as described in the *Suggested Instructional Strategies* that follow.

In grade 10 students explore the meaning of **integral exponents** (e.g. 6^{-3}) and **rational exponents** (e.g. $8^{\frac{2}{3}}$) extending to an exploration of patterns to explain negative and fraction bases and exponents (e.g. $(-8)^{\frac{1}{3}}$, $(\frac{1}{4})^{-3}$).

Rational exponents should be restricted to those for which the numerator and denominator are Natural numbers ($x^{\frac{m}{n}}$, $m, n \in N(1, 2, 3, 4, 5 \dots)$). Relate rational exponents to radicals and vice versa (e.g. $4^{\frac{1}{2}} = \sqrt{4}$, $27^{\frac{1}{3}} = \sqrt[3]{27}$, $3^{\frac{4}{2}} = (3^4)^{\frac{1}{2}} = \sqrt{3^4} = \sqrt{3^4}$).

They will establish a clear link between **numerical bases and exponents** (e.g. $2^4, 8^2$) and **literal bases and exponents** (e.g. $x^4, 8^n$) to develop and then apply the exponent laws to literal bases and exponents.

* For consistency and understanding, teachers are reminded to continue the practice established in Grade 9 of using "six to the exponent of four", or "six to the fourth" in lieu of "six to the power of four".

SCO AN3: Demonstrate an understanding of powers with integral and rational components.
[C, CN, PS, R]

ACHIEVEMENT INDICATORS

- Explain, using patterns, why $a^{-n} = \frac{1}{a^n}$, $a \neq 0$
- Explain, using patterns, why $a^{\frac{1}{n}} = \sqrt[n]{a}$, $n > 0$
- Apply the exponent laws to expressions with rational and variable bases and integral and rational exponents, and explain the reasoning :

$$(a^m)(a^n) = a^{m+n}$$

$$a^m \div a^n = a^{m-n}, a \neq 0$$

$$(a^m)^n = a^{mn}$$

$$(ab)^m = a^m b^m$$

$$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}, \quad b \neq 0$$

- Apply the exponent laws, using integral values, to evaluate expressions of the type:

$$\left(\frac{a}{b} + \frac{c}{d}\right)^m \quad \left(\frac{a}{b} - \frac{c}{d}\right)^m \quad \left(\left(\frac{a}{b}\right)^m + \left(\frac{c}{d}\right)^n\right)^k \quad \left(\left(\frac{a}{b}\right)^m - \left(\frac{c}{d}\right)^n\right)^k$$

- Express powers with rational exponents as radicals and vice versa, when m and n are natural numbers, and x is a rational number.

$$x^{\frac{m}{n}} = \left(x^{\frac{1}{n}}\right)^m = (\sqrt[n]{x})^m \quad \text{and} \quad \left(x^{\frac{m}{n}}\right)^k = \left(x^m\right)^{\frac{k}{n}} = \sqrt[n]{x^{km}}$$

- Solve a problem that involves exponent laws or radicals.
- Identify and correct errors in a simplification of an expression that involves powers.

SCO AN3: Demonstrate an understanding of powers with integral and rational components.
[C, CN, PS, R]

Suggested Instructional Strategies

- Exponent laws with reference to integral bases and whole number exponents introduced in grade 9 should be reviewed and enforced. Using the same method of exploration using patterning that was used in grade 9, teachers in grade 10 can explore integral and rational exponents.
- To introduce negative exponents students should follow the progression of the pattern first with integral bases. This can then be extended to a general rule as expressed in the exponent laws using literal expressions.

review of Grade 9	$\frac{2^4}{2^2} = 2^2$	$\frac{2 \cdot 2 \cdot 2 \cdot 2}{2 \cdot 2} = 2^2$	$\frac{2}{2} \cdot \frac{2}{2} \cdot 2 \cdot 2 = 1 \cdot 1 \cdot 4$	$\frac{16}{4} = 4$
review of Grade 9	$\frac{2^4}{2^3} = 2^1$	$\frac{2 \cdot 2 \cdot 2 \cdot 2}{2 \cdot 2 \cdot 2} = 2^1$	$\frac{2}{2} \cdot \frac{2}{2} \cdot \frac{2}{2} \cdot 2 = 1 \cdot 1 \cdot 1 \cdot 2$	$\frac{16}{8} = 2$
review of Grade 9	$\frac{2^4}{2^4} = 2^0$	$\frac{2 \cdot 2 \cdot 2 \cdot 2}{2 \cdot 2 \cdot 2 \cdot 2} = 2^0$	$\frac{2}{2} \cdot \frac{2}{2} \cdot \frac{2}{2} \cdot \frac{2}{2} = 1 \cdot 1 \cdot 1 \cdot 1$	$\frac{16}{16} = 1$
new to Grade 10	$\frac{2^4}{2^5} = 2^{-1}$	$\frac{2 \cdot 2 \cdot 2 \cdot 2}{2 \cdot 2 \cdot 2 \cdot 2 \cdot 2} = 2^{-1}$	$\frac{2}{2} \cdot \frac{2}{2} \cdot \frac{2}{2} \cdot \frac{2}{2} \cdot \frac{1}{2} = 1 \cdot 1 \cdot 1 \cdot 1 \cdot \frac{1}{2}$	$\frac{16}{32} = \frac{1}{2}$

- Use tables of examples to demonstrate to students that they can use what they already know to find missing information e.g. $2^{-3} = \frac{1}{2^3} = \frac{1}{8} = 0.125$
- As with the introduction to negative exponents, students should be given the opportunity to discover the pattern behind rational exponents. This should happen prior to directed instruction.

review of Grade 9	$9^3 \cdot 9^3$	$= (9 \cdot 9 \cdot 9)(9 \cdot 9 \cdot 9)$	$= 9^{3+3}$	$= 9^6$	$= 729 \cdot 729$	$= 531441$
	$8^3 \cdot 8^3$	$= (8 \cdot 8 \cdot 8)(8 \cdot 8 \cdot 8)$	$= 8^{3+3}$	$= 8^6$	$= 512 \cdot 512$	$= 262144$
review of Grade 9	$9^2 \cdot 9^2$	$= 9 \cdot 9 \cdot 9 \cdot 9$	$= 9^{2+2}$	$= 9^4$	$= 81 \cdot 81$	$= 6561$
	$8^2 \cdot 8^2$	$= 8 \cdot 8 \cdot 8 \cdot 8$	$= 8^{2+2}$	$= 8^4$	$= 64 \cdot 64$	$= 4096$
review of Grade 9	$9^1 \cdot 9^1$	$= 9 \cdot 9$	$= 9^{1+1}$	$= 9^2$	$= 9 \cdot 9$	$= 81$
	$8^1 \cdot 8^1$	$= 8 \cdot 8$	$= 8^{1+1}$	$= 8^2$	$= 8 \cdot 8$	$= 64$
new to Grade 10	$9^{\frac{1}{2}} \cdot 9^{\frac{1}{2}}$	$= \sqrt{9} \cdot \sqrt{9}$	$= 9^{\frac{1}{2}+\frac{1}{2}}$	$= 9^1$	$= 3 \cdot 3$	$= 9$
	$8^{\frac{1}{3}} \cdot 8^{\frac{1}{3}} \cdot 8^{\frac{1}{3}}$	$= \sqrt[3]{8} \cdot \sqrt[3]{8} \cdot \sqrt[3]{8}$	$= 8^{\frac{1}{3}+\frac{1}{3}+\frac{1}{3}}$	$= 8^1$	$= 2 \cdot 2 \cdot 2$	$= 8$

SCO AN3: Demonstrate an understanding of powers with integral and rational components.
[C, CN, PS, R]

Suggested Questions (Q) and Activities (Act) for Instruction and Assessment

Q Describe the pattern, and find the next two numbers in the sequence 1, 4, 27, 256 ...

Q A science experiment shows that the number of bacteria in a Petri dish will double every hour. If there are 1000 bacteria after 8 hours, how many will there be after:

- a) $9h$ b) $11h$ c) $14h$

Q Identify and explain the errors in the following:

1st set (whole number exponents):

- a) $4^3 + 4^2 = 4^5$ b) $\frac{x^6}{x^3} = x^2$ c) $(10^2)^5 = 10^7$ d) $\left(\frac{1}{4}\right)^2 = \frac{1}{8}$
 e) $(x - y)^3 = 3x - 3y$ f) $3^5 \times 3^2 = 3^{10}$ g) $5^3 \div 5^4 = \frac{3}{4}$

2nd set (integral exponents):

- a) $a^4 \cdot a^{-2} = a^{-8}$ b) $b^{-10} \div b^5 = b^{-5}$ c) $(c^{-3})^2 = c^{-1}$ d) $\left(\frac{2}{3}\right)^{-2} = \frac{-4}{-6}$

3rd set (rational exponents):

- a) $2^{\frac{1}{2}} \cdot 2^{\frac{1}{2}} = 2^{\frac{1}{4}}$ b) $3^{\frac{3}{4}} \div 3^{\frac{1}{4}} = 3^3$ c) $\left(4^{\frac{2}{5}}\right)^2 = 4^{\frac{2}{5}}$ d) $\left(5^{\frac{1}{2}}\right)^2 = 5^{2^{\frac{1}{2}}}$

(Note to teacher: 1st to 3rd represents a progression of difficulty from easier to harder)

Q Evaluate:

- a) $\left(\frac{2}{5} - \frac{1}{3}\right)^2$ b) $\left(\left(\frac{1}{2}\right)^3 + \left(\frac{3}{4}\right)^2\right)^2$

Q Fill in the blanks:

- a) $5^{-2} = \frac{\square}{\square}$ b) $6^{\square} = \frac{1}{6^2}$ c) $\square^{-6} = \frac{1}{10^6}$ d) $4^{-x} = \frac{1}{\square}$

(Note to teacher: Placement of answer boxes should be varied to stretch understanding)

Q Solve the following, by substituting the values given:

- a) $5x^4 + 6xy$ if $x = 2$, $y = 3$
 b) $(2x)^2$ if $x = 4$
 c) $(t + s)^{-3}$ if $t = 2$, $s = 4$

Q Indicate if the following statements are always true, sometimes true, or false. Justify your answer.

- a) The value of a power with a negative exponent is less than 0.
 b) The value of a power for which the base is a fraction is less than 1.
 c) Two powers with the exponent 0 have the same value.

SCO AN3: Demonstrate an understanding of powers with integral and rational components.
[C, CN, PS, R]

Q During an exam, three students evaluate $2^{-2} \times 2^0$ as follows:

Thomas: $2^{-2} \times 2^0 = 4^{-2} = \frac{1}{4^2} = \frac{1}{16}$

Sean: $2^{-2} \times 2^0 = 2^0 = 1$

Michel: $2^{-2} \times 2^0 = 4^0 = 1$

- a) Identify the errors made by the students.
- b) What is the correct answer? Justify your answer by explaining each step.

Act In pairs, explain how to evaluate powers such as $(-3)^{-2}$ and -3^{-2} . Compare your answers with other groups, and then as a class.

SCO AN4: Demonstrate an understanding of the multiplication of polynomial expressions (limited to monomials, binomials and trinomials), concretely, pictorially and symbolically.
[CN, R, V]

[C] Communication
Technology

[PS] Problem Solving
[V] Visualization

[CN] Connections
[R] Reasoning

[ME] Mental Math **[T]**
and Estimation

Scope and Sequence of Outcomes

Grade Nine	Grade Ten	Grade Eleven
<p>PR5 Demonstrate an understanding of polynomials (limited to polynomials of degree less than or equal to 2).</p> <p>PR6 Model, record and explain the operations of addition and subtraction of polynomial expressions, concretely, pictorially and symbolically (limited to polynomials of degree less than or equal to 2).</p> <p>PR7 Model, record and explain the operations of multiplication and division of polynomial expressions (limited to polynomials of degree less than or equal to 2) by monomials, concretely, pictorially and symbolically.</p>	<p>AN4 Demonstrate an understanding of the multiplication of polynomial expressions (limited to monomials, binomials and trinomials), concretely, pictorially and symbolically.</p>	<p>AN3: Solve problems that involve radical equations (limited to square roots). (PC11)</p> <p>AN4 Determine equivalent forms of rational expressions (limited to numerators and denominators that are monomials, binomials or trinomials).(PC11)</p> <p>AN5 Perform operations on rational expressions (limited to numerators and denominators that are monomials, binomials or trinomials).(PC11)</p> <p>AN6 Solve problems that involve rational equations (limited to numerators and denominators that are monomials, binomials or trinomials).(PC11)</p>

ELABORATION

Terminology associated with polynomials was introduced in Grade 9. Teachers should continue to use these terms in context to allow students to integrate them into their vocabulary including: *term, variable, constant, co-efficient, polynomial, degree of a term, degree of a polynomial, monomial, binomial and trinomial.*

In Grade 9, polynomials, limited to degree 1 or 2, were added and subtracted, and polynomials and monomials were multiplied and divided, concretely, pictorially and symbolically. Students multiplied powers with integer bases. They expressed polynomials with algebra tiles, with diagrams and pictures, and with symbols. Multiplication and division of polynomials was limited to polynomials with monomials.

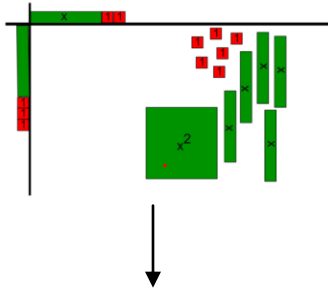
In Grade 10 multiplication of polynomials is extended to polynomials with other polynomials. The goal of this outcome is for all students to become proficient in concrete, pictorial and symbolic representation of the multiplication of polynomial expressions.

Algebra tiles and area models build an understanding of the concepts behind the symbols and should not be considered optional for students who are able to master the more traditional symbolic models without these tools. Although there will be a greater reliance on symbolic representations as they progress into higher grades, ability with concrete and pictorial representations will help to build a deeper understanding of the concepts and the applications. Emphasis should be placed on the ability to switch between alternate representations, leading to a proficiency in symbolic representation.

SCO AN4: Demonstrate an understanding of the multiplication of polynomial expressions (limited to monomials, binomials and trinomials), concretely, pictorially and symbolically.
 [CN, R, V]

The following illustrates the multiplication of the polynomial: $(x + 2)(x + 3)$, expressed concretely, pictorially, and symbolically.

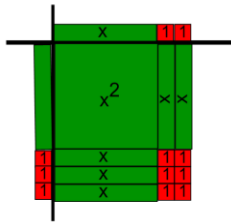
** be sure to review the values of each tile and how those tiles fit together to create the area*



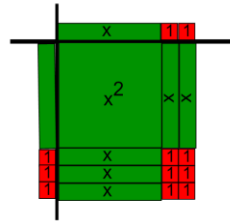
	x	2
x	$x \cdot x = x^2$	$x \cdot 2 = 2x$
3	$3 \cdot x = 3x$	$(2)(3) = 6$

$$\begin{aligned} (x + 2)(x + 3) &= x^2 + 2x + 3x + 6 \\ &= x^2 + 5x + 6 \end{aligned}$$

$$\begin{aligned} (x + 2)(x + 3) &= x^2 + 3x + 2 + 6 \\ &= x^2 + 5x + 6 \end{aligned}$$



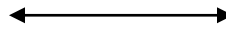
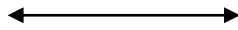
$$= x^2 + 5x + 6$$



concrete
(with algebra tiles)

pictorial
(on paper)

symbolic



SCO AN4: Demonstrate an understanding of the multiplication of polynomial expressions (limited to monomials, binomials and trinomials), concretely, pictorially and symbolically.
[CN, R, V]

ACHIEVEMENT INDICATORS

- Model the multiplication of two given binomials, concretely or pictorially, and record the process symbolically.
- Relate the multiplication of two binomial expressions to an area model.
- Explain, using examples, the relationship between the multiplication of binomials and the multiplication of two-digit numbers.
- Verify a polynomial product by substituting numbers for the variables.
- Multiply two polynomials symbolically, and combine like terms in the product.
- Generalize and explain a strategy for multiplication of polynomials.
- Identify and explain errors in a solution for a polynomial multiplication.

Suggested Instructional Strategies

- Present students with a selection of *binomial x binomial* expressions with the product included. Ask students to find the pattern or the relationship between the questions and their product. Have students discover a strategy on their own by showing them repeated examples until they identify the pattern. Once they have come up with a strategy, have them test it to make sure it applies to a multitude of examples.
- After having students solve expressions, have them verify their solutions through substitution:

e.g. $(x + 2)(x + 3) = x^2 + 5x + 6$

to verify, substitute $x = 1$

<i>left side</i>	<i>right side</i>
$(x + 2)(x + 3)$	$x^2 + 5x + 6$
$= (1 + 2)(1 + 3)$	$= 1^2 + 5(1) + 6$
$= (3)(4)$	$= 1 + 5 + 6$
$= 12$	$= 12$
- Present solutions to surface area problems that include errors and have students identify the errors and correct them.

SCO AN4: Demonstrate an understanding of the multiplication of polynomial expressions (limited to monomials, binomials and trinomials), concretely, pictorially and symbolically.
[CN, R, V]

Suggested Questions (Q) and Activities (Act) for Instruction and Assessment

Q Use the distributive property to multiply the following polynomials with algebra tiles, pictorially, and symbolically. Verify the product by substituting the variable with a number value:

- a) $2(y + 3)$ b) $3b(4 + 2b)$ c) $2(x^2 + 5x + 4)$ d) $(x + 4)(x + 3)$
e) $(3x + 2)(x)$ f) $(5 + x)(2x + 1)$ g) $(6 + 2y)(1 + y)$ h) $(2x + 3)(x + 9)$

Act Have students write an expression to determine the floor area of an L-shaped building, having given each side length a monomial or binomial value. Have students verify the product by substituting numerical values for the variable.

Q A classmate has missed the lesson on multiplying binomials. How would you explain to him the process used to determine the product of two binomials?

SCO AN5: Demonstrate an understanding of common factors and trinomial factoring, concretely, pictorially and symbolically.
[C, CN, R, V]

[C] Communication
[T] Technology

[PS] Problem Solving
[V] Visualization

[CN] Connections
[R] Reasoning

[ME] Mental Math
and Estimation

Scope and Sequence of Outcomes

Grade Nine	Grade Ten	Grade Eleven
<p>PR5 Demonstrate an understanding of polynomials (limited to polynomials of degree less than or equal to 2).</p> <p>PR6 Model, record and explain the operations of addition and subtraction of polynomial expressions, concretely, pictorially and symbolically (limited to polynomials of degree less than or equal to 2).</p> <p>PR7 Model, record and explain the operations of multiplication and division of polynomial expressions (limited to polynomials of degree less than or equal to 2) by monomials, concretely, pictorially and symbolically.</p>	<p>AN5 Demonstrate an understanding of common factors and trinomial factoring concretely, pictorially and symbolically.</p>	<p>RF2 Demonstrate an understanding of the characteristics of quadratic functions including vertex, intercepts, domain and range, axis of symmetry (<i>FM11</i>)</p> <p>AN4: Determine equivalent forms of rational expressions (limited to numerators and denominators that are monomials, binomials or trinomials). (<i>PC11</i>)</p> <p>AN5: Perform operations on rational expressions (limited to numerators and denominators that are monomials, binomials or trinomials). (<i>PC11</i>)</p> <p>AN6: Solve problems that involve rational equations (limited to numerators and denominators that are monomials, binomials or trinomials). (<i>PC11</i>)</p> <p>RF1 Factor polynomial expressions of the form</p> <ul style="list-style-type: none"> • $ax^2 + bc + c, a \neq 0$ • $a^2x^2 - b^2y^2, a \neq 0, b \neq 0$ • $a(f(x))^2 + b(f(x))^2 + c, a \neq 0$ • $a^2(f(x))^2 - b^2(g(y))^2, a \neq 0, b \neq 0$ <p>where a, b, and c are rational numbers. (<i>PC11</i>)</p> <p>RF5: Solve problems that involve quadratic equations. (<i>PC11</i>)</p>

ELABORATION

The concept of factoring of whole numbers was introduced earlier in this unit in AN1, and the multiplication of binomials and trinomials covered in AN4.

For this outcome students will develop and understanding of common factors and trinomial factoring of the form $ax^2 + bx + c$ where a, b and $c \in I$. For purposes of differentiation $a = 1$ or $a > 1$. Trinomial factoring will include perfect squares and the difference of squares.

Exploration of common factors and trinomial factoring should begin with factoring as the reverse of multiplication using algebra tiles and the area model. There should be a progression from concrete to pictorial to symbolic representations to build an understanding of the concepts behind the symbols. Mastering concrete and pictorial representations should not be considered optional for students who are able to master symbolic models without these tools.

SCO AN5: Demonstrate an understanding of common factors and trinomial factoring, concretely, pictorially and symbolically.
[C, CN, R, V]

ACHIEVEMENT INDICATORS

- Determine the common factors in the terms of a polynomial, and express the polynomial in factored form.
- Model the factoring of a trinomial, concretely or pictorially, and record the process symbolically.
- Factor a polynomial that is a difference of squares, and explain why it is a special case of trinomial factoring where $b = 0$.
- Identify and explain errors in a polynomial factorization.
- Factor a polynomial, and verify by multiplying the factors.
- Explain, using examples, the relationship between multiplication and factoring of polynomials.
- Generalize and explain strategies used to factor a trinomial.
- Express a polynomial as a product of its factors.

Suggested Instructional Strategies

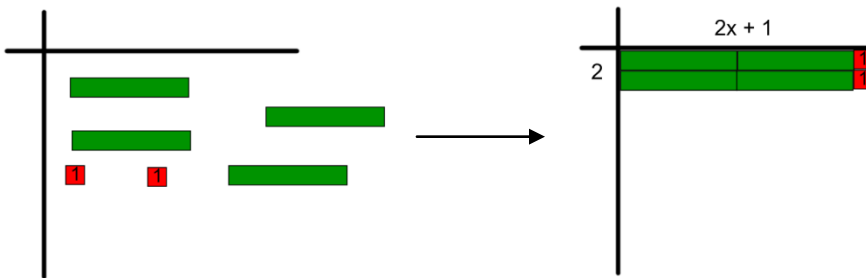
- To encourage deep understanding, it is recommended that there is a progression from concrete to pictorial to symbolic representations. Complexity should be gradually increased. For example, begin with binomials with positive values, extend to binomials with negative values, then mixed values, and finally, use trinomials with coefficients as your highest level of understanding.
- As opposed to explicitly teaching students that factoring is the opposite of the distributive property, encourage students to come up with their own strategy through the use of repeated examples. Be sure to have students test their strategy to ensure it works with various examples. When introducing a difference of two squares, again, have students discover the rule as opposed to explicitly teaching it.
- Algebra placemats can be laminated and provided as individual workspace for students when working with algebra tiles. Alternatively or as well, laminated cardstock can be used with dry erase markers (red and green) to visually represent work, in place of using algebra tiles.

SCO AN5: Demonstrate an understanding of common factors and trinomial factoring, concretely, pictorially and symbolically.
[C, CN, R, V]

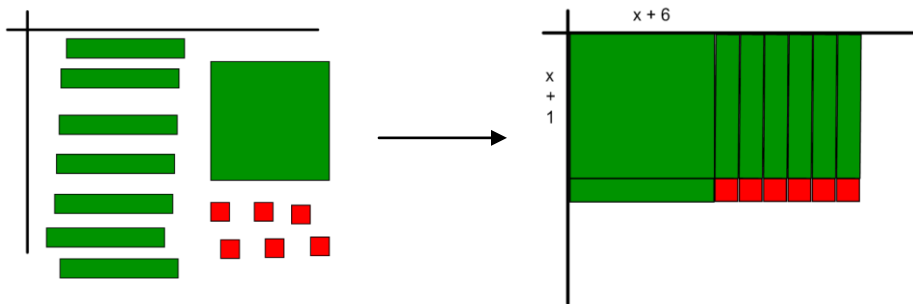
- Factoring can be introduced as the reverse of multiplication using algebra tiles, and the area model. Starting with the product to be factored, algebra tiles can be arranged as a rectangle, the dimensions of which will be the factors.

To factor a polynomial using an algebra placemat, students will assemble the tiles into a rectangular shape. The factors are the dimensions of the rectangle. Where there is more than one solution, this rectangle could be arranged differently.

e.g. To factor $4x + 2$, arrange tiles on the placemat as a rectangle of dimensions 2 by $2x + 1$. The sides of the rectangle are its factors so: $4x + 2 = 2(2x + 1)$.



e.g. To factor $x^2 + 7x + 6$, arrange tiles on the placemat as a rectangle of dimensions $(x + 1)$ by $(x + 6)$. The sides of the rectangle are its factors so:
 $x^2 + 7x + 6 = (x + 1)(x + 6)$



SCO AN5: Demonstrate an understanding of common factors and trinomial factoring, concretely, pictorially and symbolically.
[C, CN, R, V]

Suggested Questions (Q) and Activities (Act) for Instruction and Assessment

Q Fill in the blanks:

a) $12x + 18y = (\square)(2x + 3y)$

b) $3x^2 - 5x = (\square)(3x - 5)$

c) $4ab + 3ac = (\square)(4b + 3c)$

d) $3y^2 + 18y = 3y(y + \square)$

e) $14a - 12b = 2(\square - 6b)$

f) $x^2 + 6x + \square = (x + \square)(x + \square)$

g) $x^2 + \square x + 12 = (x + \square)(x + \square)$

h) $x^2 - \square x + 5 = (x - \square)(x - \square)$

i) $x^2 - \square x - 12 = (x - \square)(x + \square)$

j) $2x^2 - x - 6 = (2x + \square)(x - \square)$

k) $\square x^2 + \square x + \square = (3x + 1)(2x + 5)$

(Note to teacher: a) to k) represents increasing levels of difficulty.)

Act Find two different trinomials that both have $(x + 3)$ as a factor. Verify your answer by substituting the value 1 for the variable

(Note to teacher: Repeat this activity with a variety of factors.)

Q Factor the following expressions. Verify your answer by substituting the value 1 for the variable.

a) $x^2 - 9$

b) $12x^2 - 13x + 3$

c) $9x^2 - 4y^2$

d) $y^2 - 16$

e) $1 - 64t^2$

f) $x^2 + 6x + 9$

g) $15 - x - 2x^2$

h) $4m^2 - 25$

SCO **RF1: Interpret and explain the relationships among data, graphs and situations.**
[C, CN, R, T, V]

[C] Communication
[T] Technology

[PS] Problem Solving
[V] Visualization

[CN] Connections
[R] Reasoning

[ME] Mental Math
and Estimation

Relations and Functions

Scope and Sequence of Outcomes

Grade Nine	Grade Ten	Grade Eleven
PR2 Graph linear relations, analyze the graph and interpolate or extrapolate to solve problems.	RF1 Interpret and explain the relationships among data, graphs and situations.	<p>RF1 Model and solve problems that involve systems of linear inequalities in two variables (<i>FM11</i>)</p> <p>RF2 Demonstrate an understanding of the characteristics of quadratic functions, including vertex, intercepts, domain and range, axis of symmetry. (<i>FM11</i>)</p> <p>RF2: Graph and analyze absolute value functions (limited to linear and quadratic functions) to solve problems. (<i>PC11</i>)</p> <p>RF4: Analyze quadratic functions of the form $y = ax^2 + bx + c$ to identify characteristics of the corresponding graph, including vertex, domain and range, direction of opening, axis of symmetry, x- and y-intercepts, and to solve problems (<i>PC11</i>)</p> <p>RF5: Solve problems that involve quadratic equations. (<i>PC11</i>)</p>

ELABORATION

Throughout middle school, students investigated plotting points on graphs and on a Cartesian plane, as well as interpolating and extrapolating from a given graph.

In Grade 7, students studied the concept of central tendency and were introduced to range. In Grade 10, this outcome now introduces the concept of the domain and extends their understanding of range. Students should be given a variety of opportunities to develop their critical thinking skills as they determine which numbers are reasonable in a given situation including examples of discrete and continuous data.

Students will interpret data given to them in various forms such as table of values, graphs or real life situations. They will be able to sketch a graph from a set of data or for a given situation, and conversely will be able to describe a situation given a graph.

Discrete Data: Data is discrete when values have a finite or limited number of possible values e.g. number of students in a class, number of tickets sold, or how many items were purchased. The plotted points are not joined together.

Continuous Data: Data is continuous when values have an infinite number of possible values within a selected range e.g. temperature or time. The plotted points are joined together.

For a given relationship, students will determine restrictions on the **domain** (the set of all independent variables or *x values*) and the **range** (the set of all dependent variables or *y values*).

To help students develop their understanding of linear relationships they should use both technology, and paper and pencil.

SCO RF1: Interpret and explain the relationships among data, graphs and situations.
[C, CN, R, T, V]

ACHIEVEMENT INDICATORS

- Graph, with and without technology, a set of data, and determine the restrictions on the domain and range.
- Explain why data points should or should not be connected on the graph for a situation (discrete versus continuous data).
- Describe a possible situation for a given graph.
- Sketch a possible graph for a given situation.
- Determine, and express in a variety of ways, the domain and range of a graph, a set of ordered pairs, and a table of values.

Suggested Instructional Strategies

- Demonstrate an understanding of how graphs and data are related to life, have students explain the situation that a graph represents, as well as graph the data when given an explanation of a particular situation.
- Rather than using x and y , emphasis should be given to labeling the axes of the graph to represent the given situation.
- Provide students with a variety of problems which use both discrete and continuous data.
- Students should be provided an opportunity to explain situations to their peers to show their understanding.
- Domain and range must be explored through data, graphs and given situations. Students should understand which numbers are “reasonable” in a given situation.
- Students should be exposed to various forms of graphing using technology such as graphing calculators, and other computer aided software.

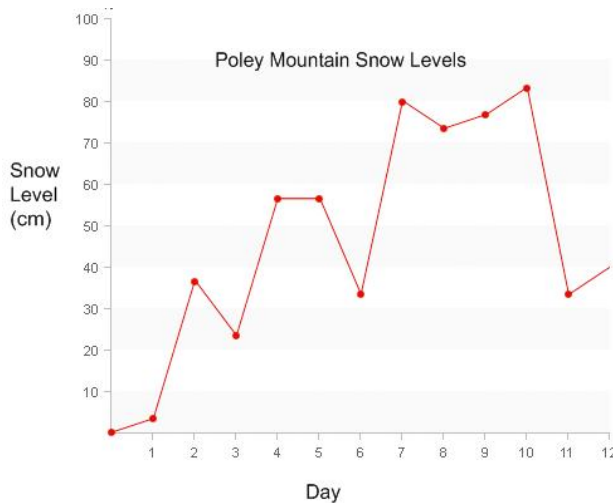
SCO RF1: Interpret and explain the relationships among data, graphs and situations.
[C, CN, R, T, V]

Suggested Questions (Q) and Activities (Act) for Instruction and Assessment

Act Present students with a situation, or have them imagine one for which they would require a series of photographs. Then have them complete the following:
A photographer charges a sitting fee of \$20 and \$1.50 for every photograph ordered.

- a) Graph the situation using at least 5 points.
- b) Explain why the points are not connected.
- c) Is it possible that a person would be charged \$30? Justify your solution.

Q Interpret the following graph:



Explain what is happening to the snow levels between given days. For example, what could have happened between day 5 and 6? Have students explore various possibilities to explain changes.

Q For each of the following situations do a rough sketch of the relation and include a reasonable domain and range.

- a) When you turn on a hot water faucet the temperature of the water depends on how many seconds the water has been running. Sketch a graph of temperature vs. time.
- b) Your height depends on your age. Sketch a graph of height vs. age.
- c) The number of Christmas cards a greeting card store sells depends on the time of year. Sketch a graph of the number of cards sold versus the month of the year.
- d) You put some ice cubes in a glass and fill it with cold water on a summer day. Sketch a graph of the temperature of the water versus the time it is sitting on a table.
- e) The time of sunset depends on the time of the year. Sketch a graph of the time of sunset versus time of the year.

SCO RF2: Demonstrate an understanding of relations and functions.
[C, R, V]

[C] Communication [T] Technology	[PS] Problem Solving [V] Visualization	[CN] Connections [R] Reasoning	[ME] Mental Math and Estimation
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Scope and Sequence of Outcomes

Grade Nine	Grade Ten	Grade Eleven
<p>PR1: Generalize a pattern arising from a problem-solving context using linear equations and verify by substitution.</p> <p>PR2: Graph linear relations, analyze the graph and interpolate or extrapolate to solve problems.</p>	<p>RF2 Demonstrate an understanding of relations and functions.</p>	<p>RF2 Demonstrate an understanding of the characteristics of quadratic functions, including vertex, intercepts, domain and range, axis of symmetry. <i>(FM11)</i></p> <p>RF2: Graph and analyze absolute value functions (limited to linear and quadratic functions) to solve problems. <i>(PC11)</i></p> <p>RF3: Analyze quadratic functions of the form $y = a(x - p)^2 + q$ and determine the vertex, domain and range, direction of opening, axis of symmetry, x- and y-intercepts. <i>(PC11)</i></p> <p>RF4: Analyze quadratic functions of the form $y = ax^2 + bx + c$ to identify characteristics of the corresponding graph, including vertex, domain and range, direction of opening, axis of symmetry, x- and y-intercepts, and to solve problems <i>(PC11)</i></p>

ELABORATION

This will be an introduction to the concept of **relations** and **functions**. Given a graph or a table of values students should be able to determine and explain the difference between a relation and a function.

Relation: For every value of x in a relation there is at least one value of y

Function: For every value of x there is only one y value.

Relations can be represented in various forms as shown below. For example:

Examples of Functions:

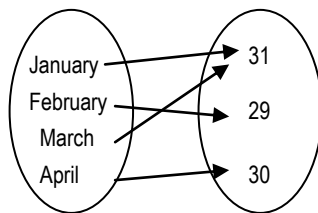
Table

Points on a graph

<i>x</i>	<i>y</i>
2	4
3	6
4	10
5	10
6	10

Arrow Diagram

days in the month



Set of Ordered Pairs

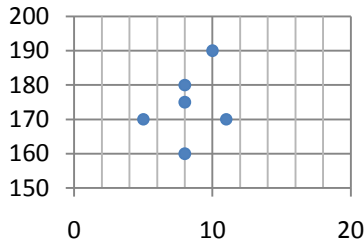
vehicle that has this # wheels

$\{(unicycle, 1), (bicycle, 2), (motorcycle, 2), (tricycle, 3), (car, 4)\}$

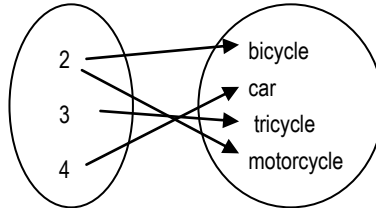
SCO RF2: Demonstrate an understanding of relations and functions.
[C, R, V]

Examples that are NOT Functions

Graph
shoe size for each height



Arrow Diagram
wheels on a vehicle



List of ordered pairs
name and home of
students at a workshop

{(Marie, Ottawa),
(Sam, Toronto),
(Matthew, Halifax),
(Carolyn, Bathurst),
(Matthew, Rivière-du-Loup)}

ACHIEVEMENT INDICATORS

- Explain, using examples, why some relations are not functions but all functions are relations.
- Determine if a set of ordered pairs represents a function.
- Sort a set of graphs as functions or non-functions.
- Generalize and explain rules for determining whether graphs and sets of ordered pairs represent functions.

Suggested Instructional Strategies

- Provide students with two tables - one of functions, and one with relations that are not functions. Have them graph each and discuss similarities and differences they notice. This could be used as a small group discussion or a whole group discussion.
- Students should be introduced to the vertical line test as a means of testing if graphs are functions and expand this rule to analyze a table of values.
- Tables of functions, and non-functions can be photocopied and distributed among students to discuss and determine which they are.

SCO RF2: Demonstrate an understanding of relations and functions.
[C, R, V]

Suggested Questions (Q) and Activities (Act) for Instruction and Assessment

Act Below are two tables. The first table lists four data sets of relations that are functions and the second table four data sets of relations that are not functions. Express each of the relations as a graph, as an arrow diagram, and as a set of ordered pairs and describe to a partner what you would look for to determine if it is a function or not for each representation.

Relations that are Functions

<i>x</i>	<i>y</i>	<i>x</i>	<i>y</i>	<i>x</i>	<i>y</i>	<i>x</i>	<i>y</i>
2	1	1	2	0	-2	1	7
4	2	2	4	1	4	2	-1
6	7	3	6	2	-3	3	2
8	3	4	8	3	5	4	6
10	4	5	10	4	6	5	4

Relations that are not Functions

<i>x</i>	<i>y</i>	<i>x</i>	<i>y</i>	<i>x</i>	<i>y</i>	<i>x</i>	<i>y</i>
1	2	2	1	3	-2	1	7
2	4	4	2	1	4	5	-1
3	6	2	7	2	-3	3	2
4	8	8	3	3	5	4	6
1	10	10	4	4	6	5	4

Act With real life examples, have students create two relations to share with a partner, each with a different format (table, arrow diagram, graph, set of ordered pairs). One must be a function and one a non-function. The partner must then explain which relation is the function and which is not.

SCO RF3 Demonstrate an understanding of slope with respect to: rise and run; line segments and lines; rate of change; parallel lines; and perpendicular lines.
[PS, R, V]

[C] Communication Technology	[PS] Problem Solving [V] Visualization	[CN] Connections [R] Reasoning	[ME] Mental Math [T] and Estimation
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Scope and Sequence of Outcomes

Grade Nine	Grade Ten	Grade Eleven
	RF3 Demonstrate an understanding of slope with respect to: rise and run; line segments and lines; rate of change; parallel lines; and perpendicular lines.	<p>A1: Solve problems that require the manipulation and application of formulas related to slope and rate of change, Rule of 72, finance charges, the Pythagorean theorem and trigonometric ratios. <i>(FWM11)</i></p> <p>A2: Demonstrate an understanding of slope. <i>(FWM11)</i></p> <p>PR1: Solve problems that involve the application of rates. <i>(FM11)</i></p>

ELABORATION

This is the first formal opportunity students have had to study slope in a math course. However, slope or steepness is a common day-to-day concept, and students may also have been introduced to slope in science class.

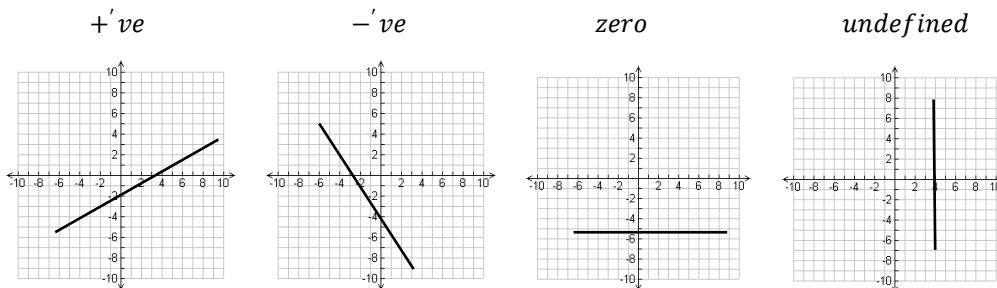
For this outcome students should understand and be proficient at using the different methods of finding slope.

On a graph, slope can be represented by $\frac{\text{rise}}{\text{run}}$, or the $\frac{\text{vertical change}}{\text{horizontal change}}$.

As a rate of change slope can be represented as $\frac{\text{change in } y}{\text{change in } x}$.

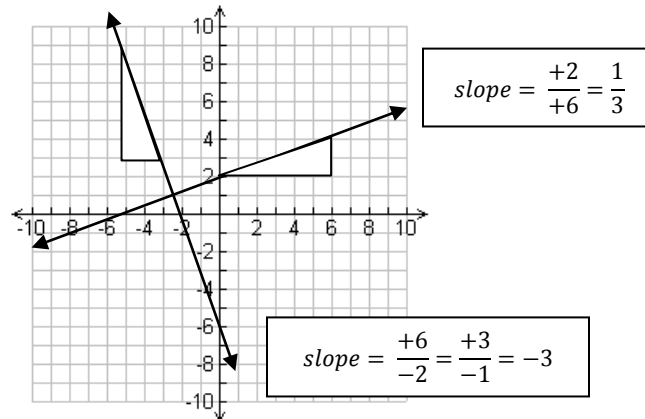
As an algorithm slope can be represented as $\frac{(y_2 - y_1)}{(x_2 - x_1)}$.

On graphs students should quickly be able to identify slopes as positive, negative, zero, and undefined. For a given line segment on a graph, slope will be constant for the full length.



SCO RF3 Demonstrate an understanding of slope with respect to: rise and run; line segments and lines; rate of change; parallel lines; and perpendicular lines.
[PS, R, V]

Slopes of parallel lines are equal. Slopes of perpendicular lines are negative reciprocals. For example:



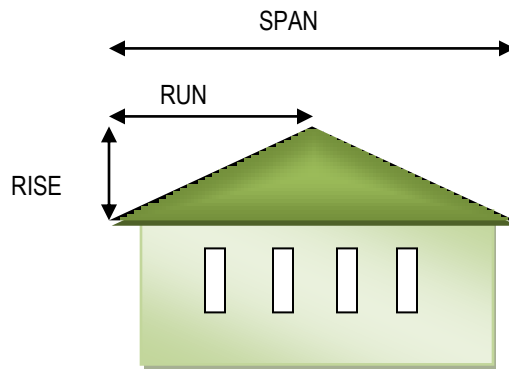
ACHIEVEMENT INDICATORS

- Determine the slope of a line segment by measuring or calculating the rise and run.
- Classify lines in a given set as having positive or negative slopes.
- Explain the meaning of the slope of a horizontal or vertical line.
- Explain why the slope of a line can be determined by using any two points on that line.
- Explain, using examples, slope as a rate of change.
- Draw a line, given its slope and a point on the line.
- Determine another point on a line, given the slope and a point on the line.
- Generalize and apply a rule for determining whether two lines are parallel or perpendicular.
- Solve a contextual problem involving slope.

SCO RF3 Demonstrate an understanding of slope with respect to: rise and run; line segments and lines; rate of change; parallel lines; and perpendicular lines.
[PS, R, V]

Suggested Instructional Strategies

- Provide photos of slopes in everyday life. For example the grade of land, ski hill, roof of a building, wheel chair ramp, stairway, etc. Ask students why slope is important. The relationship of rise to vertical and run to horizontal can be explored here leading to the overall concept of slope being “rise/run”.

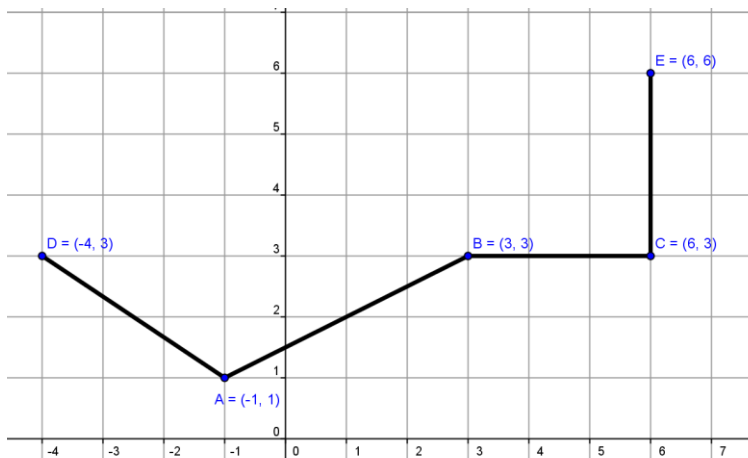


- Provide a grid with various slopes labeled. Have students calculate the individual slopes and discuss the concept of positive, negative and undefined slopes (see suggested activities below).
- Give three lines that are parallel and have students find and compare the slope of each given the formula. Then provide three sets of lines that are perpendicular again have students find and compare the slope of each set. Have students make predictions about the slope of parallel and perpendicular lines.
- Have students draw a line given the slope. Compare between students to demonstrate that not all lines will be in the same position on the graph

SCO RF3 Demonstrate an understanding of slope with respect to: rise and run; line segments and lines; rate of change; parallel lines; and perpendicular lines.
[PS, R, V]

Suggested Questions (Q) and Activities (Act) for Instruction and Assessment

Q Calculate the individual slopes for each line segment. Which segment has a positive slope, a negative slope, a slope of 0, and undefined.



Act As a homework assignment, have students accompany a family member who is driving on an errand from home and back again (someone else should be driving). Set the trip counter and record the distance and location every 5 minutes. Have students plot time versus distance from home. **

Questions:

- What was the slope of each 5 minute interval of the trip? How does this relate to speed?
- What was the most common speed?.
- How fast were you going on each leg of the trip?
- Are there any negative slopes? If so, what does this indicate?
- Are there any slopes of zero? If so, what does this indicate?
- Why are there no undefined slopes? Would this be possible? Explain.

** Challenge students to design and carry out alternative methods for doing this exercise on foot, by bicycle, or busing. This could give a positive spin to this exercise for those students who do not use a car for financial or environmental reasons.

SCO RF4 Describe and represent linear relations, using: words; sets of ordered pairs; tables of values; graphs; and equations.
[C, CN, R, V]

[C] Communication Technology	[PS] Problem Solving [V] Visualization	[CN] Connections [R] Reasoning	[ME] Mental Math [T] and Estimation
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Scope and Sequence of Outcomes

Grade Nine	Grade Ten	Grade Eleven
<p>PR1 Generalize a pattern arising from a problem-solving context using linear equations and verify by substitution.</p>	<p>RF4 Describe and represent linear relations, using: words; sets of ordered pairs; tables of values; graphs; and equations.</p>	<p>RF2 Demonstrate an understanding of the characteristics of quadratic functions, including vertex, intercepts, domain and range, axis of symmetry. <i>(FM11)</i></p> <p>RF2: Graph and analyze absolute value functions (limited to linear and quadratic functions) to solve problems. <i>(PC11)</i></p>

ELABORATION

Since Grade 7, students have solved contextual problems using linear equations, and have represented linear data in a variety of forms such as tables, graphs, equations or situations.

In Grade 10, students will determine if a relationship is linear or not, by using a variety of methods e.g. common differences, graph shape, degree of the equation, creating a table of values. They will learn to identify which is the independent variable and which is the dependent variable. They will find the slope in each of the various forms studied and recognize that in any linear relationship the slope is constant.

Students should be able to determine if a situation would provide linear data. For example: running a bath is linear, but the spreading of a virus or increasing car speed from a stop sign are non-linear.

ACHIEVEMENT INDICATORS

- Identify independent and dependent variables in a given context.
- Determine whether a situation represents a linear relation, and explain why or why not.
- Determine whether a graph represents a linear relation, and explain why or why not.
- Determine whether a table of values or a set of ordered pairs represents a linear relation, and explain why or why not.
- Draw a graph from a set of ordered pairs within a given situation, and determine whether the relationship between the variables is linear.
- Determine whether an equation represents a linear relation, and explain why or why not.
- Match corresponding representations of linear relations.

SCO RF4 Describe and represent linear relations, using: words; sets of ordered pairs; tables of values; graphs; and equations.
[C, CN, R, V]

Suggested Instructional Strategies

- Provide students with data from various situations and have them determine which relations are linear and which are non-linear. Have them explain what this means.
- Provide the students with one of the following: an equation, a set of ordered pairs or table of values, a graph, a word description of a situation. Then have the students create the other three. For example:

Given the equation $y = 3x - 5$ create a table of values or a set of ordered pairs, a graph, and a word description of a situation described by the equation.

- Give students a list of situations and relationships and provide time for them to work together to determine if the situation would be a linear or non-linear relation. For example: the relationship between step length and distance travelled is linear, but the relationship between length of sides and area of a square is non-linear.
- Have students compare linear equations and non-linear equations. Provide linear and non-linear equations and have students complete a table of values as well as a graph for each. For example: equations could include:

$$y = x + 2, \quad y = 6x - 7, \quad y = x, \quad y = x^2 + 2x + 1, \quad y = x^2.$$

Technology may also be used to illustrate the graphs.

- Have students access data from the newspaper or the internet e.g. stocks from the newspaper, and collect data and graph it over a period of time. Use this graph to discuss relations, and determine linear and/ or non-linear aspects. Teachers could explore the slope of the line of best fit as an extension.

SCO RF4 Describe and represent linear relations, using: words; sets of ordered pairs; tables of values; graphs; and equations.
[C, CN, R, V]

Suggested Questions (Q) and Activities (Act) for Instruction and Assessment

Act Using the Stock page from the newspaper, have each student pick a stock company of their choice, and then record and graph the value of the stock over a one month period. The stocks page can be posted on the wall and the internet can be used to access the TSX.

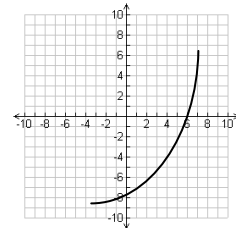
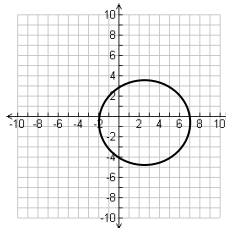
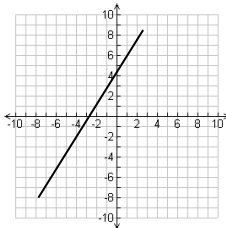
Have students use the graph to determine:

- Linear and/or non-linear sections
- The slope of the line of best fit for the linear sections (determined by eye)

Have students discuss what factors might affect the value of the stock, and relate this to current events.

(Alternatively students could look at the history of that stock over the previous month and extrapolate data to determine if they should invest in this stock by studying slopes).

Q Which of the following relations are linear?



Time (months)	Height (inches)
0	22
6	23
12	25
18	27
24	32

Price	Taxes
\$10	\$1.50
\$60	\$9.00
\$110	\$16.50
\$160	\$24.00
\$210	\$31.50

Q Which of the following relations are linear?

- a) $\{(2,10), (4,15), (6, 20), (8,25), (10,30), (12,35)\}$
- b) $\{(0,1), (20,2), (40,4), (60,8), (80,1), (100,32)\}$
- c) $x^2 - 5x + 3 = y$
- d) $x + 5 = 13$
- e) $y = 23$
- f) $5 + x^3 = 2x + 1$

SCO **RF5: Determine the characteristics of the graphs of linear relations, including the: intercepts; slope; domain; and range.**
[CN, PS, R, V]

[C] Communication [T] Technology	[PS] Problem Solving [V] Visualization	[CN] Connections [R] Reasoning	[ME] Mental Math and Estimation
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Scope and Sequence of Outcomes

Grade Nine	Grade Ten	Grade Eleven
PR2 Graph linear relations, analyze the graph and interpolate or extrapolate to solve problems.	RF5 Determine the characteristics of the graphs of linear relations, including the: intercepts; slope; domain; and range.	<p>RF2 Demonstrate an understanding of the characteristics of quadratic functions, including vertex, intercepts, domain and range, axis of symmetry. <i>(FM11)</i></p> <p>RF3: Analyze quadratic functions of the form $y = a(x - p)^2 + q$ and determine the vertex, domain and range, direction of opening, axis of symmetry, x- and y-intercepts. <i>(PC11)</i></p> <p>RF4: Analyze quadratic functions of the form $y = ax^2 + bx + c$ to identify characteristics of the corresponding graph, including vertex, domain and range, direction of opening, axis of symmetry, x- and y-intercepts, and to solve problems <i>(PC11)</i></p> <p>RF5: Solve problems that involve quadratic equations. <i>(PC11)</i></p>

ELABORATION

Students had experience interpolating or extrapolating information to solve problems using linear relations in Grade 9. Students should also have an understanding of slope (RF3) and domain and range (RF1).

For this outcome students will identify and express **intercepts** as values or ordered pairs (e.g., $y = 2$ or $(0,2)$) as well as explain what they represent when given the graph. They will learn to characterize the slope as positive, negative, zero or undefined. Students will graph linear relations when given one intercept and slope, or when given both intercepts.

The y -axis can be defined as the line that has an infinite number of y -intercepts, and has the equation $x = 0$. The x -axis can be defined as the line that has an infinite number of x -intercepts, and has the equation $y = 0$.

With this outcome, students become familiar with the standard notation for domain and range. For domain this is e.g. $\{x \mid -3 \leq x \leq 6, x \in \mathbf{R}\}$ and read as: “ x such that x is greater than or equal to -3 and less than or equal to 6 , and is a member of the Real number system”. For range this is e.g. $\{y \mid -2 \leq y \leq 4, y \in \mathbf{R}\}$ and is read as y such that y is greater than or equal to -2 and less than or equal to 4 , and is a member of the Real number system”.

SCO RF5: Determine the characteristics of the graphs of linear relations, including the: intercepts; slope; domain; and range.
[CN, PS, R, V]

ACHIEVEMENT INDICATORS

- Determine the intercepts of the graph of a linear relation, and state the intercepts as values or ordered pairs.
- Determine the slope of the graph of a linear relation.
- Determine the domain and range of the graph of a linear relation.
- Sketch a linear relation that has one intercept, two intercepts or an infinite number of intercepts.
- Identify the graph that corresponds to a given slope and y -intercept.
- Determine the slope and y -intercept that correspond to a given graph.
- Solve a contextual problem that involves intercepts, slope, domain or range of a linear relation.

Suggested Instructional Strategies

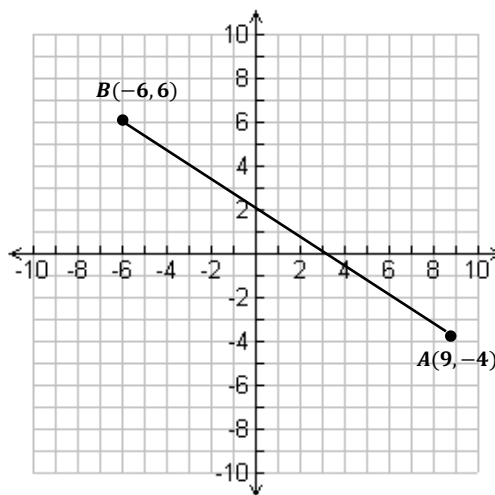
- At this point a quick review of slope formulas would be advisable. Students could be presented with graphs of linear equations and be required to state the slopes, intercepts, domain and range. When expressing domain and range that contain end points students should express domain and range in the following format:

$$\text{Domain} = \{x \mid -6 \leq x \leq 9, x \in R\}$$

$$\text{Range} = \{y \mid -4 \leq y \leq 6, y \in R\}$$

The x -intercept can be expressed as $x = 3$ or $(3,0)$

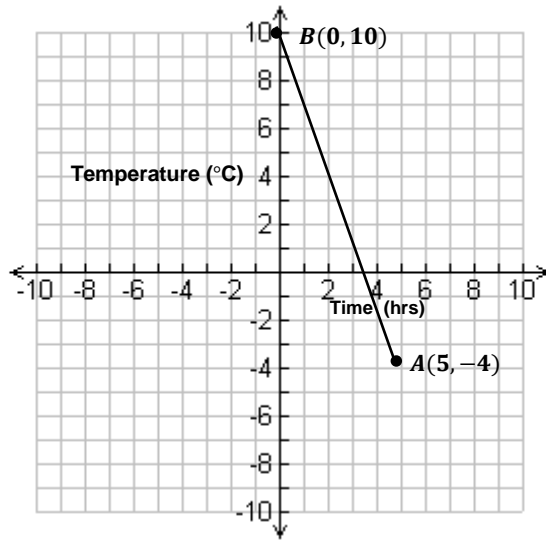
The y -intercept can be expressed as $y = 2$ or $(0,2)$



SCO RF5: Determine the characteristics of the graphs of linear relations, including the: intercepts; slope; domain; and range.
[CN, PS, R, V]

Suggested Questions (Q) and Activities (Act) for Instruction and Assessment

Q The following graph represents the temperature of sea water placed in a freezer. Explain the meaning of the slope, what *Point A* represents, what the x and y -intercepts represent, and the domain and range.



Note to teacher: The slope represents the rate at which the temperature is decreasing. The y -intercept represents the temperature of the water when it is placed in the freezer. The x -intercept indicates the temperature after 4 hours. Point A represents the time and temperature at which the water begins to freeze (sea water freezes at $\sim 4^{\circ}\text{C}$.)

SCO **RF6: Relate linear relations expressed in: slope-intercept form ($y = mx + b$), general form ($Ax + By + C = 0$), and slope-point form ($y - y_1 = m(x - x_1)$), to their graphs.**
[CN, R, T, V]

[C] Communication Technology [PS] Problem Solving [V] Visualization [CN] Connections [R] Reasoning [ME] Mental Math [T] and Estimation

Scope and Sequence of Outcomes

Grade Nine	Grade Ten	Grade Eleven
<p>PR2: Graph linear relations, analyze the graph and interpolate or extrapolate to solve problems.</p>	<p>RF6 Relate linear relations expressed in slope-intercept form ($y = mx + b$), general form ($Ax + By + C = 0$), and slope-point form ($y - y_1 = m(x - x_1)$), to their graphs.</p>	<p>RF1 Model and solve problems that involve systems of linear inequalities in two variables. (FM11)</p> <p>RF2 Demonstrate an understanding of the characteristics of quadratic functions, including vertex, intercepts, domain and range, axis of symmetry. (FM11)</p> <p>RF3: Analyze quadratic functions of the form $y = a(x - p)^2 + q$ and determine the vertex, domain and range, direction of opening, axis of symmetry, x- and y-intercepts. (PC11)</p> <p>RF4: Analyze quadratic functions of the form $y = ax^2 + bx + c$ to identify characteristics of the corresponding graph, including vertex, domain and range, direction of opening, axis of symmetry, x- and y-intercepts, and to solve problems. (PC11)</p>

ELABORATION

Linear equations can be rewritten into several different forms. This outcome provides an introduction to three forms of equations for linear relations, or a straight line. Students will learn to identify all three forms of the equation and to change from one form to another.

Students will graph linear relations from each of the three forms of equations. They should be given the opportunity to discover these concepts and strategies through investigations.

The **slope-intercept form** for a linear relation is $y = mx + b$ where m is the slope of the line and b is the y -intercept. Vertical lines, having undefined slope, are not represented by this form. Using the slope-intercept form, the linear relation can be graphed using the slope (m) and the y -intercept (b).

For example : $y = 2x + 6$

- the y -intercept is 6, or (0,6)
- the slope is $2 = \frac{2}{1} = \frac{6}{3}$, so to find a second point, start at the intercept, go over 3 units and up 6 units to (3,12)

Draw a line through (0,6) and (3,12) for the linear function.

SCO **RF6: Relate linear relations expressed in: slope-intercept form ($y = mx + b$), general form ($Ax + By + C = 0$), and slope-point form ($y - y_1 = m(x - x_1)$), to their graphs.**
[CN, R, T, V]

The **general form** for a linear relation is: $Ax + By + C = 0$ where A and B are not both $= 0$. The equation is usually written so that $A \geq 0$, by convention. If $A \neq 0$, then the x -intercept, is $-\frac{C}{A}$. If $B \neq 0$, then the y -intercept is $-\frac{C}{B}$ and the slope of the line is $-\frac{A}{B}$. Using the general form the linear relation can be drawn by joining the intercepts, or by joining the intercept and one other point.

For example: $2x + 4y + 6 = 0$

To draw the line by joining the intercepts:

- the x -intercept is $-\frac{C}{A} = -\frac{6}{2} = -3$, or $(-3, 0)$
- the y -intercept is $-\frac{C}{B} = -\frac{6}{4} = -1.5$, or $(0, -1.5)$

Draw a line through $(-3, 0)$ and $(0, -1.5)$ for the linear function.

To draw the line by joining an intercept and one point:

- the x -intercept is $-\frac{C}{A} = -\frac{6}{2} = -3$, or $(-3, 0)$
- the slope is $-\frac{A}{B} = -\frac{2}{4}$ so start at the intercept and go over 4 (*add 4 to x*) and down 2 (*subtract 2 from y*) to $(1, -2)$

Draw a line through $(-3, 0)$ and $(1, -2)$ for the linear function.

The **slope-point form** for a linear equation is $(y - y_1) = m(x - x_1)$ where m is the slope of the line and (x_1, y_1) is any point on the line. The point-slope form expresses the fact that the difference in the y coordinate between two points on a line ($y - y_1$) is proportional to the difference in the x coordinate ($x - x_1$). The proportionality constant is m (the slope of the line). Using the slope-point form, the linear relation is drawn by using (x_1, y_1) and the slope (m).

For example : $(y - 6) = 2(x - 3)$

To draw the line by joining two points:

- one point on the line is (x_1, y_1) or $(6, 3)$
- the slope is $2 = \frac{2}{1}$, so to find a second point, start at $(6, 3)$ and go over 1 (*add 1 to x_1*), and up 2 (*add 2 to y_1*), to $(7, 5)$

Draw a line through $(6, 3)$ and $(7, 5)$ for the linear function.

SCO RF6: Relate linear relations expressed in: slope-intercept form ($y = mx + b$), general form ($Ax + By + C = 0$), and slope-point form ($y - y_1 = m(x - x_1)$), to their graphs.
[CN, R, T, V]

ACHIEVEMENT INDICATORS

- Express a linear relation in slope-intercept, general, and slope-point forms, and compare the graphs.
- Generalize and explain strategies for graphing a linear relation in slope–intercept, general or slope-point form.
- Graph, with and without technology, a linear relation given in slope–intercept, general or slope–point form.
- Identify equivalent linear relations from a set of linear relations.
- Match a set of linear relations to their graphs.

Suggested Instructional Strategies

- Have students graph the same linear equation in all three forms by finding two points from the equations given. This will show the same graph can be expressed using three different types of equations. Have students discuss and determine which form of the equation is easiest in creating the graph.

Suggested Questions (Q) and Activities (Act) for Instruction and Assessment

Q Three forms of the same equation are:

$y = 2x + 3$, $2x - y + 3 = 0$, and $(y - 7) = 2(x - 2)$. Graph each equation.

Q In a group of three people, identify each person as a “slope-intercept form“, a “slope-point form“, or a “general form“. Each person creates an equation in his or her form and then draws the graph of that equation. Pass your equation and graph to the person on your right. That person will now write the equation in their form. Pass this one more time to complete all three forms for each equation. Check each others as a group to ensure it has been done correctly.

Switch each person’s responsibility to another equation form twice more and repeat the process, until everyone has had a chance to practice each form. Check answers as a group and hand in the results.

SCO **RF7: Determine the equation of a linear relation given a graph, a point and the slope, two points, a point and the equation of a parallel or perpendicular line, and a scatter plot.**
[CN, PS, R, V]

[C] Communication Technology	[PS] Problem Solving [V] Visualization	[CN] Connections [R] Reasoning	[ME] Mental Math [T] and Estimation
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Scope and Sequence of Outcomes

Grade Nine	Grade Ten	Grade Eleven
<p>PR1: Generalize a pattern arising from a problem-solving context using linear equations and verify by substitution.</p>	<p>RF7 Determine the equation of a linear relation, given a graph, a point and the slope, two points, a point and the equation of a parallel or perpendicular line, and a scatter plot.</p>	<p>RF3: Analyze quadratic functions of the form $y = a(x - p)^2 + q$ and determine the vertex, domain and range, direction of opening, axis of symmetry, x- and y-intercepts. (PC11)</p> <p>RF4: Analyze quadratic functions of the form $y = ax^2 + bx + c$ to identify characteristics of the corresponding graph, including vertex, domain and range, direction of opening, axis of symmetry, x- and y-intercepts, and to solve problems. (PC11)</p>

ELABORATION

Students have learned to identify and match linear equations with the respective graph, and to find slope and points from graphs. This outcome introduces students to writing equations using the **slope-point form** of linear equations, when given either a graph from which they will identify a point on the line and the slope, or when given points on the graph and/or the slope directly.

For a scatter plot the line of best fit will be the linear relation which most closely describes the data. The equation of this line and its correlation to the date should be determined using technology (e.g. graphics calculator or other technology). Within this contexts students should discuss and explore the concept of correlation and how it varies.

ACHIEVEMENT INDICATORS

- Determine the slope and y-intercept of a given linear relation from its graph, and write the equation in the form $y = mx + b$.
- Write the equation of a linear relation, given its slope and the coordinates of a point on the line, and explain the reasoning.
- Write the equation of a linear relation, given the coordinates of two points on the line, and explain the reasoning.
- Write the equation of a linear relation, given the coordinates of a point on the line and the equation of a parallel or perpendicular line, and explain the reasoning.
- Graph linear data generated from a context, and write the equation of the resulting line.
- Determine the equation of the line of best fit from a scatter plot using technology and discuss the correlation
- Solve a contextual problem, using the equation of a linear relation.

SCO	RF7: Determine the equation of a linear relation given a graph, a point and the slope, two points, a point and the equation of a parallel or perpendicular line, and a scatter plot. [CN, PS, R, V]
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Suggested Instructional Strategies

- To determine the equation of a linear relation, students should practice various ways of finding a point and the slope, depending on the information given. For example:

To determine a point on the line:

- Select a point from the graph.
- The student is given the x and y coordinates of a point on the line.
- An x or y intercept is determined from a given equation by substituting either $x=0$ (for the y-intercept) or $y=0$ (for the x-intercept).
- A table of values for the line is provided to the student from which any point can be selected.

To determine the slope of the line:

- A value for slope is given directly.
 - Any two points on the line are selected and the rise and run between the points is determined to give the slope.
 - The x and y coordinates for two points are given, and the slope formula is used to calculate the slope.
 - A parallel line is given, and the slope is determined from this line.
 - A perpendicular line is given, and its slope determined. The negative reciprocal is the slope of the line.
 - If the line is horizontal the slope is zero. If the line is vertical the slope is undefined.
- Depending on the information given, students can express the equation of the linear relation in slope-intercept, general, or slope-point form. Students should be encouraged to determine which form best suits a given situation.
 - Scatter plots are one of three topics covered in the Pearson WNCP Manitoba Curriculum Supplement available as a free download from:
http://www.pearsoncanadaschool.com//media/canada/math10wncp_manitoba_curriculum_companion_final.pdf
 - For exploring scatter plots the following site provides an interactive program which allows students to quickly plot points and explore the effect on the r value, as they plot points closer to and further from the line of best fit.
<http://www.shodor.org/interactivate/activities/Regression/>

SCO	RF7: Determine the equation of a linear relation given a graph, a point and the slope, two points, a point and the equation of a parallel or perpendicular line, and a scatter plot. [CN, PS, R, V]
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Suggested Questions (Q) and Activities (Act) for Instruction and Assessment

Q Mary wanted to go on a student exchange to Mexico but she only had \$56 dollars in her bank account. She got a job walking dogs in her neighbourhood for \$50 a week.

If she saves every penny she makes, write the equation to best represent the total amount of money she will have in her bank account after a certain amount of weeks. Using this equation determine how many weeks it will take her to save \$2000.

Extension: Mary's friend started with \$250 and was paid the same amount. Calculate the number of weeks it would take her to save \$2000.

(note to teacher: This is a good place to discuss discrete and continuous data and whether a linear relation could represent both.)

SCO **RF8: Solve problems that involve the distance between two points and the midpoint of a line segment.**
[C, CN, PS, T, V]

[C] Communication [T] Technology	[PS] Problem Solving [V] Visualization	[CN] Connections [R] Reasoning	[ME] Mental Math and Estimation
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Scope and Sequence of Outcomes

Grade Nine	Grade Ten	Grade Eleven
	RF8: Solve problems that involve the distance between two points and the midpoint of a line segment.	T2: Solve problems, using the three primary trigonometric ratios for angles from 0° to 360° in standard position. (PC11)

ELABORATION

Students will develop both the **Distance Formula** and **Midpoint Formula** by building on their former knowledge and understanding of xy coordinates on a Cartesian plane which they were introduced to in Grade 6, and on their understanding of the Pythagorean Theorem to solve problems Grades 8 and 9. Teachers should take care to develop a clear understanding of these as distinct concepts as experience has shown that students frequently confuse these two formulas.

Developing an understanding of the Distance Formula to determine distance between two points will build on students' understanding of the Pythagorean Theorem. Explorations can begin with determining the length of the special cases of horizontal or vertical lines on a Cartesian plane, in which the distance is the difference between the two x values (horizontal) or the two y values (vertical). The Distance formula is the general form of a rearrangement of the Pythagorean formula for finding the length of the hypotenuse. The formula for the distance between two points A and B is:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Development of an understanding of the midpoint formula is based on a different concept – that of the mean of two values. On a Cartesian plane the midpoint of a line is found by determining the mean of the x and the y coordinates of the two end points of the line. If the endpoints are $P(x_1, y_1)$ and $Q(x_2, y_2)$ the coordinates for the midpoint will be the mean between the two x values and the mean between the two y values, or

$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right).$$

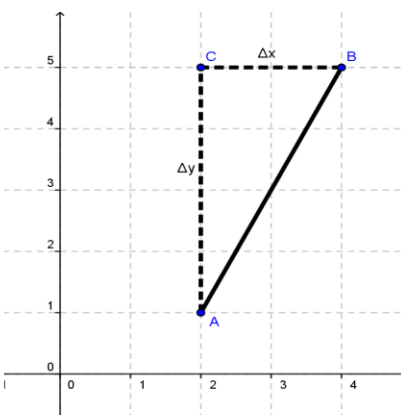
SCO **RF8: Solve problems that involve the distance between two points and the midpoint of a line segment.**
[C, CN, PS, T, V]

ACHIEVEMENT INDICATORS

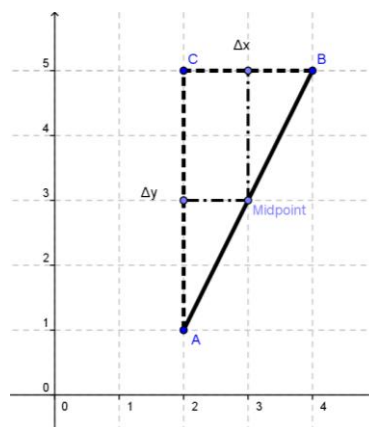
- Determine the distance between two points on a Cartesian plane using a variety of strategies.
- Determine the midpoint of a line segment, given the endpoints of the segment, using a variety of strategies..
- Determine an endpoint of a line segment, given the other endpoint and the midpoint, using a variety of strategies.
- Solve a contextual problem involving distance between two points or midpoint of a line segment.

Suggested Instructional Strategies

- The length and midpoint of a line are two of three topics covered in the Pearson WNCP Manitoba Curriculum Supplement available as a free download from:
http://www.pearsoncanadaschool.com//media/canada/math10wncp_manitoba_curriculum_companion_final.pdf
- Students should be given a chance to develop formulas, rather than being given them to start.
- With their knowledge of Pythagorean theorem students should be able to determine the length of a line segment on grid paper such as the figure shown below. Section 4.2, Book 3 of Mathematical Modeling (old resource) can be used as a supplemental resource for this unit.
- A similar diagram can be used to develop an understanding of the Midpoint Formula in a specific case and then for the general case, using the mean of x coordinates and the mean of the y coordinates.



$$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$



$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

SCO	RF8: Solve problems that involve the distance between two points and the midpoint of a line segment. [C, CN, PS, T, V]
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Suggested Questions (Q) and Activities (Act) for Instruction and Assessment

- Q** The line segment AB is formed by joining the ordered pairs A(-4,3) and B(2, -3).
a) Determine the length of the line segment AB
b) find the coordinated of the Midpoint AB.
- Q** A triangle has the vertices A(-3,1), B(1,7), and C(5,1).
a) Find the perimeter
b) Classify the triangle as scalene, isosceles or equilateral.
- Q** Show that points P(5,-1), Q(2,8) and R(-2,0) lie on a circle whose centre is C(2,3).
- Q** One endpoint of a line segment is (-4,3). The Midpoint is (-3,6), Find the other endpoint.
- Q** Given two points, A and B, find the point 1/3 of the way from A to B.

SCO RF9: Represent a linear function, using function notation. [CN, ME, V]								
<table style="width: 100%; border: none;"> <tr> <td style="width: 25%; border: none;">[C] Communication</td> <td style="width: 25%; border: none;">[PS] Problem Solving</td> <td style="width: 25%; border: none;">[CN] Connections</td> <td style="width: 25%; border: none;">[ME] Mental Math and Estimation</td> </tr> <tr> <td style="border: none;">[T] Technology</td> <td style="border: none;">[V] Visualization</td> <td style="border: none;">[R] Reasoning</td> <td style="border: none;"></td> </tr> </table>	[C] Communication	[PS] Problem Solving	[CN] Connections	[ME] Mental Math and Estimation	[T] Technology	[V] Visualization	[R] Reasoning	
[C] Communication	[PS] Problem Solving	[CN] Connections	[ME] Mental Math and Estimation					
[T] Technology	[V] Visualization	[R] Reasoning						

Scope and Sequence of Outcomes

Grade Nine	Grade Ten	Grade Eleven
PR1: Generalize a pattern arising from a problem-solving content using linear equations and verify by substitution.	RF9: Represent a linear function, using function notation.	RF outcomes: Function notation required for all RF outcomes (<i>FM11, PC11</i>)

ELABORATION

This outcome provides students with an introduction to **function notation** for linear functions. A function gives each input value (x) a unique corresponding output value (y). The $f(x)$ notation can be thought of as another way of representing the y -value.

Students should make a connection between the input and output values and ordered pairs as visualized on a graph. For example, the notation $f(2) = 5$ indicates that the point with coordinates $(2,5)$ lies on the graph of $f(x)$.

In function notation such as $f(x)$, the “ f ” is an arbitrary name, but g , and h are commonly used, as in $g(x)$ and $h(x)$. The letter in the parentheses indicates the independent variable used when the function is represented by an equation. For example, writing $A(r) = \pi r^2$ indicates “ A ” is the name of the function and “ r ” is the independent variable.

Students will be expected to express an equation in two variables using function notation. For example, the equation $y = 4x - 1$ can be written as $f(x) = 4x - 1$. Conversely, they will express an equation in function notation as a linear function in two variables. For example, $h(t) = -3t + 1$ can be written as $h = -3t + 1$. Often the function name is related to a context as in this example, in which the function name is $h(t)$, for a problem which involves the height (h) of an object at a certain time (t).

Students will determine the range value given the domain value. For example, students given $f(x) = 5x - 7$, determine $f(1)$. Conversely, students will determine the domain value given the range value. For example, for the function $f(x) = 4x - 1$, where $f(x) = 3$, students will solve for x for $4x - 1 = 3$.

The input for a function can be another function. For example, students could be asked to determine the simplified expression for $f(x + 1)$ if $f(x) = 3x - 5$.

SCO RF9: Represent a linear function, using function notation.
[CN, ME, V]

ACHIEVEMENT INDICATORS

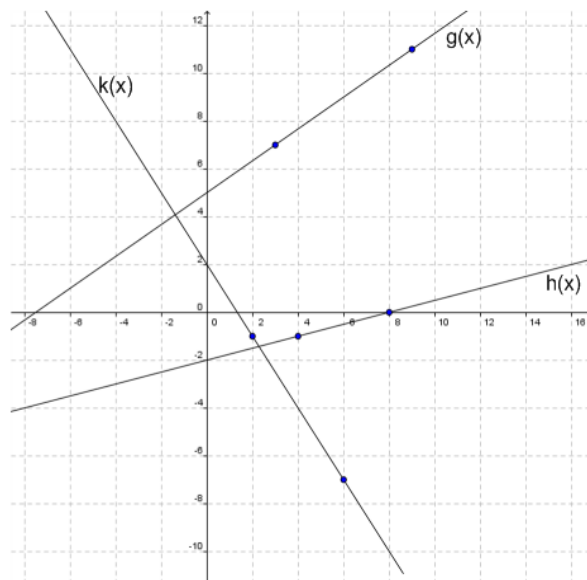
- Express the equation of a linear function in two variables, using function notation.
- Express an equation given in function notation as a linear function in two variables.
- Determine the related range value, given a domain value for a linear function; e.g., if $f(x) = 3x - 2$, determine $f(-1)$.
- Determine the related domain value, given a range value for a linear function; e.g., if $g(t) = 7 + t$, determine t so that $g(t) = 15$.
- Sketch the graph of a linear function expressed in function notation.

Suggested Instructional Strategies

- A review of rearranging equations would be helpful since function notation requires the linear relation to be solved for the y -variable.
- A common student error occurs when the parenthesis is mistakenly used as multiplication in function notation. For example, $f(4) = 8$ does not mean $4f = 8$. It is important for students to make the connection that this is a place holder which represents the domain value.
- Another error occurs when students are given a function such as $h(t) = 4t - 3$ where $h(t) = 18$ and asked to determine t . Students often substitute the given value for the independent variable rather than the dependent variable. This would be a good opportunity to reiterate the purpose and meaning of function notation.

Suggested Questions (Q) and Activities (Act) for Instruction and Assessment

Q Write the equation for each function, $g(x)$, $h(x)$ and $k(x)$. Then calculate the value of $g(15)$, $h(28)$ and $k(x) = 17$.



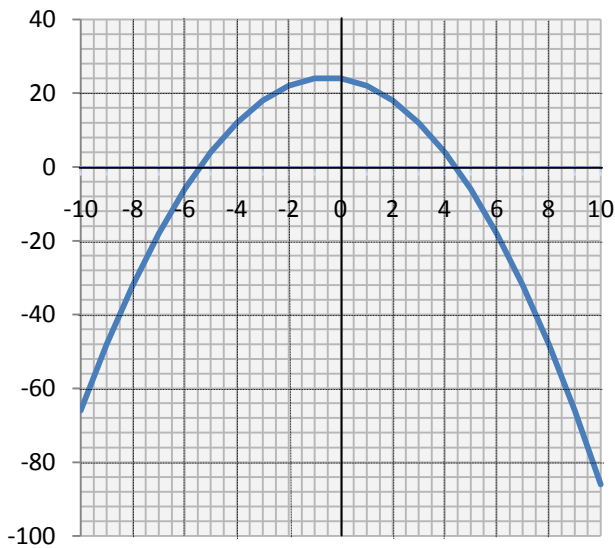
SCO RF9: Represent a linear function, using function notation.
[CN, ME, V]

Q Evaluate the following:

- a) $d(t) = 3t + 4$, determine $d(3)$
- b) $f(x) = x^2 - 2x - 24$, determine $f(-2)$
- c) $h(t) = 4t^2 - 3t$, determine $h(1) + h(-2)$
- d) $f(x) = 5x - 11$, find the value of x that makes $f(x) = 9$
- e) $g(x) = -2x + 5$, find the value of x that makes $g(x) = -7$

Q Given the function $f(x)$, find:

- c) $f(-2.5)$
- d) $f(2)$
- e) x such that $f(x) = -40$



Q The perimeter of a rectangle is $P = 2l + 2w$. If it is known that the length must be 6 ft , then the perimeter is a function of the width. Write this function using function notation.

SCO RF10 Solve problems that involve systems of linear equations in two variables, graphically and algebraically.
[CN, PS, R, T, V]

[C] Communication [T] Technology	[PS] Problem Solving [V] Visualization	[CN] Connections [R] Reasoning	[ME] Mental Math and Estimation
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Scope and Sequence of Outcomes

Grade Nine	Grade Ten	Grade Eleven
<p>PR2: Graph linear relations, analyze the graph and interpolate or extrapolate to solve problems.</p> <p>PR3 Model and solve problems using linear equations of the form: $ax = b$; $\frac{x}{a} = b$, $a \neq 0$; $ax + b = c$; $\frac{x}{a} + b = c$, $a \neq 0$; $ax = b + cx$; $a(x + b) = c$; $ax + b = cx + d$; $a(bx + c) = d(ex + f)$; $\frac{a}{x} = b$, $x \neq 0$ where a, b, c, d, e and f are rational numbers.</p>	<p>RF10 Solve problems that involve systems of linear equations in two variables, graphically and algebraically.</p>	<p>RF1 Model and solve problems that involve systems of linear inequalities in two variables. (FM11)</p> <p>RF6 Solve, algebraically and graphically, problems that involve systems of linear-quadratic and quadratic-quadratic equations in two variables. (PC11)</p> <p>RF7 Solve problems that involve linear and quadratic inequalities in two variables. (PC11)</p>

ELABORATION

When two lines intersect, the coordinates of the point of intersection is the solution of the linear system. Students will model situations using a system of linear equations and conversely, describe a possible situation which is modeled by a given linear system. They will learn both graphical and algebraic methods for solving systems of linear equations.

Students will translate a word problem into a system of linear equations and will solve the problem by graphing, verifying the solution. They should be comfortable solving systems of linear equations graphically both with and without technology. Emphasis should be placed on real life applications such as cell phone providers or cab companies.

In previous outcomes, students graphed linear equations using the slope-intercept method, the slope-point form, and using the x and y intercept method. Identifying the form of the equation will help students decide which method they should choose when graphing the lines.

However, there are limitations to solving a linear system by graphing. For example, non-integral intersection points may be difficult to determine exactly and the solution will be an estimate of the coordinates of the point of intersection. For exact answers algebraic methods can be used instead.

Students will solve systems of linear equations algebraically using substitution and elimination. After determining the solution to the linear system, they will verify the solution by direct substitution or by graphing.

Systems of linear equations can have different numbers of solutions. By graphing the linear system, students can determine if the system of linear equations has one solution (intersecting lines), no solution (parallel lines) or an infinite number of solutions (coincident lines).

Students can also use the slope and y-intercept of each equation to determine the number of solutions of the linear system. When the slopes of the lines are different, the lines intersect at

SCO RF10 Solve problems that involve systems of linear equations in two variables, graphically and algebraically.
[CN, PS, R, T, V]

one point and the system has one solution. When the slopes of the lines are the same but the y -intercept is different, the two lines will never intersect and the system has no solution. When the slopes of the lines and the y -intercept is the same, the two lines are identical and the system has an infinite number of solutions.

The elimination method will also indicate the number of solutions of a linear system. When a unique value for x and y can be determined, the system of equations has one solution. When adding equations eliminates the x variable resulting in a false statement such as, for example, $0x + 0y = 27$, there are no solutions and the lines must be parallel. When the result is $0x + 0y = 0$, for any value of x or y , this statement is true, and there are an infinite number of solutions of the linear system, indicating that the lines are coincident.

Linear systems can be solved both graphically or algebraically. It is important that students be able to identify which method is most efficient.

ACHIEVEMENT INDICATORS

- Model a situation, using a system of linear equations.
- Relate a system of linear equations to the context of a problem.
- Determine and verify the solution of a system of linear equations graphically, with and without technology.
- Explain the meaning of the point of intersection of a system of linear equations.
- Determine and verify the solution of a system of linear equations algebraically.
- Explain, using examples, why a system of equations may have no solution, one solution or an infinite number of solutions.
- Explain a strategy to solve a system of linear equations.
- Solve a problem that involves a system of linear equations.

Suggested Instructional Strategies

- To solve a system of linear equations using graphing technology students may need to rearrange equations to functional form. Also, it may be necessary to review domain and range to help students to determine appropriate settings for the viewing window.
- Provide students an opportunity to decide which algebraic method is more efficient when solving a linear system by focusing on the coefficients of like variables. If necessary, rearrange the equations so that like variables appear in the same position in both equations. Substitution may be more efficient if the coefficient of a term is 1. Elimination, on the other hand, may be more efficient if the variable in both equations have the same or opposite coefficient.
- Have students compare two companies that provide the same service or product. They should create an equation to represent each service and then graph the relationships for both companies on the same graph. Include examples in which the systems of equations do not intersect, and are parallel.

SCO RF10 Solve problems that involve systems of linear equations in two variables, graphically and algebraically.
[CN, PS, R, T, V]

Students should then discuss which service would be best considering various situations and, at the point at which lines intersect discuss if the advantages of one plan over another change. e.g. A comparison of cell phone plans and different usage can be made, considering that one person may text mostly and only use his or her phone sporadically, and another person may use his or her phone continuously and seldom text.

If the equations do not intersect, have students discuss what this means in terms of what service is less expensive e.g. Two taxi companies may have different base rates, but then may charge the same amount per kilometre travelled. The company with the lower base rate would always be less expensive.

Suggested Questions (Q) and Activities (Act) for Instruction and Assessment

Q For the following situations create two equations, and use these equations to solve the problems algebraically.

- a) At a high school hockey game, students paid \$4 per ticket and adults paid \$6. The number of students who attended was 300 more than the number of adults. If the total of all ticket sales was \$2400, how many of the attendees were students and how many were adults?
- b) Discount Taxi charges \$2 per kilometre with an initial fee of \$4. In the same community T's Taxi charges \$2 per kilometre with a flat rate of \$5. Create equations for each taxi company and graph both on the same graph. Reflect on the fact that the two equations have the same slope and the two lines are parallel.
- c) George has a beautiful farm in Miramichi, NB where he raises Emus and Buffalo. On his farm there are 64 legs and 20 heads. How many of each animal does he have?

Act Have students explain and demonstrate the meaning of having systems of equations that are parallel, by having them create and solve their own problem.

Act Ask students to describe or model a situation that cannot be modeled by a linear system and explain why a linear system is unsuitable. *

Q John bought 8 books. Some books cost \$13 each and the rest of the books cost \$24 each. He spent a total of \$209. Write a system of linear equations that could represent this situation. *

Q Create a situation relating to coins that can be modelled by the linear system shown below and explain the meaning of each variable. *

$$x + y = 24$$

$$0.25x + 0.05y = 4.50$$

Q Mitchell solved the linear system $2x + 3y = 6$ and $x - 2y = -6$. His solution was (2,4). Verify whether Mitchell's solution is correct. Explain how Mitchell's results can be illustrated on a graph. *

SCO RF10 Solve problems that involve systems of linear equations in two variables, graphically and algebraically.
[CN, PS, R, T, V]

Q Jill earns \$40 plus \$10 per hour. Tony earns \$50 plus \$5 per hour. Graphically represent the linear system relating Jill and Tony's earnings. Identify the solution to the linear system and explain what it represents. *

Q Explain which system you would prefer to solve, without using technology *

$$\left. \begin{array}{l} y = \frac{9}{2}x - \frac{23}{2} \\ y = \frac{2}{11}x - \frac{16}{11} \end{array} \right\} \qquad \left. \begin{array}{l} y = -\frac{2}{5}x - \frac{2}{5} \\ y = -\frac{3}{7}x - \frac{33}{70} \end{array} \right\}$$

Act Explain to another student how you would solve the following system of linear equations.

$$\left. \begin{array}{l} 4x - 7y = -39 \\ 3x + 5y = -19 \end{array} \right\}$$

Justify the method you chose.*

Q A test has twenty questions worth 100 points. The test consists of selected response questions worth 3 points each and constructed response worth 11 points each. How many selected response questions are on the test? *

Q The cost of a buffet dinner for a family of six was \$48.50 (\$11.75 per adult, \$6.25 per child). How many members paid each price? *

Act Have students to create a linear system that could be solved more efficiently through:

- substitution rather than using elimination or graphing. Solve the system.
- elimination rather than using substitution or graphing. Solve the system. *

Act Have students research the cost of renting a car in St. John's for a single day.

- Ask students how the study of linear relations and linear systems would help you decide which company to rent from.
- What is the effect of distance travelled?
- What other variable(s) effect the cost of renting a car? *

Q a) Solve the linear system using elimination.

$$\left. \begin{array}{l} 2x - 5y = 10 \\ 4x - 10y = 20 \end{array} \right\}$$

- What does the solution tell you about the nature of the lines of the equations?
- Convert the equations to slope-intercept form to confirm this conclusion. *

Act *Chutes and Ladders*: Each group of four students is given a game board, 1 die, 1 pack of *system of equations* cards and 4 disks to move along the board. Ask students to draw a card and roll the die. If the roll is a 1, the student will solve the system by graphing. If they roll a 2 or 5, they solve the system by substitution. If they roll a 3 or 4, they solve the system by elimination. If the roll is a 6, they solve by a method of their choice. If the system is solved correctly, the student moves the number of spaces on the board that corresponds to the roll on the die. If the answer is incorrect, the person on the left has the opportunity to answer the question and move on the board.*

* Copied or adapted with permission from NL Mathematics 1201 Curriculum Guide available at: <http://www.ed.gov.nl.ca/edu/k12/curriculum/guides/mathematics/math1201/unit7.pdf>

SUMMARY OF CURRICULUM OUTCOMES

Number, Relations and Functions 10

[C] Communication [PS] Problem Solving [CN] Connections [R] Reasoning
[ME] Mental Mathematics and Estimation [T] Technology [V] Visualization

Algebra and Number

General Outcome: Develop algebraic reasoning and number sense.

Specific Outcomes

- AN1** Demonstrate an understanding of factors of whole numbers by determining the prime factors, greatest common factor, least common multiple, square root, cube root. [CN, ME, R]
- AN2** Demonstrate an understanding of irrational numbers by representing, identifying and simplifying irrational numbers, ordering irrational numbers. [CN, ME, R, V]
- AN3** Demonstrate an understanding of powers with integral and rational exponents. [C, CN, PS, R]
- AN4** Demonstrate an understanding of the multiplication of polynomial expressions (limited to monomials, binomials and trinomials), concretely, pictorially and symbolically. [CN, R, V]
- AN5** Demonstrate an understanding of common factors and trinomial factoring, concretely, pictorially and symbolically. [C, CN, R, V]

Relations and Functions

General Outcome: Develop algebraic and graphical reasoning through the study of relations.

Specific Outcomes

- RF1** Interpret and explain the relationships among data, graphs and situations. [C, CN, R, T, V]
- RF2** Demonstrate an understanding of relations and functions. [C, R, V]
- RF3** Demonstrate an understanding of slope with respect to rise and run, line segments and lines, rate of change, parallel lines, perpendicular lines. [PS, R, V]
- RF4** Describe and represent linear relations, using words, ordered pairs, tables of values, graphs, equations. [C, CN, R, V]
- RF5** Determine the characteristics of the graphs of linear relations, including the intercepts, slope, domain, range. [CN, PS, R, V]
- RF6** Relate linear relations expressed in: slope–intercept form ($y = mx + b$), general form ($Ax + By + C = 0$), slope–point form ($y - y_1 = m(x - x_1)$). [CN, R, T, V]
- RF7** Determine the equation of a linear relation, given: a graph, a point and the slope, two points, a point and the equation of a parallel or perpendicular line, a scatter plot. [CN, PS, R, V]
- RF8** Solve problems that involve the distance between two points and the midpoint of a line segment. [C, CN, PS, T, V]
- RF9** Represent a linear function, using function notation. [CN, ME, V]
- RF10** Solve problems that involve systems of linear equations in two variables, graphically and algebraically. [CN, PS, R, T, V]

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