Geometry, Measurement and Finance 10 Curriculum

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New Brunswick Nouveau

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Curriculum Overview for Grades 10-12 Mathematics

BACKGROUND AND RATIONALE

Mathematics curriculum is shaped by a vision which fosters the development of mathematically literate students who can extend and apply their learning and who are effective participants in society.

It is essential the mathematics curriculum reflects current research in mathematics instruction. To achieve this goal, *The Common Curriculum Framework for Grades 10–12 Mathematics: Western and Northern Canadian Protocol* has been adopted as the basis for a revised mathematics curriculum in New Brunswick. The Common Curriculum Framework was developed by the seven ministries of education (Alberta, British Columbia, Manitoba, Northwest Territories, Nunavut, Saskatchewan and Yukon Territory) in collaboration with teachers, administrators, parents, business representatives, post-secondary educators and others.

The framework identifies beliefs about mathematics, general and specific student outcomes, and achievement indicators agreed upon by the seven jurisdictions. This document is based on both national and international research by the WNCP and the NCTM.

There is an emphasis in the New Brunswick curriculum on particular key concepts at each grade which will result in greater depth of understanding and ultimately stronger student achievement. There is also a greater emphasis on number sense and operations concepts in the early grades to ensure students develop a solid foundation in numeracy.

The intent of this document is to clearly communicate high expectations for students in mathematics education to all education partners. Because of the emphasis placed on key concepts at each grade level, time needs to be taken to ensure mastery of these concepts. Students must learn mathematics with understanding, actively building new knowledge from experience and prior knowledge (NCTM Principles and Standards, 2000).

BELIEFS ABOUT STUDENTS AND MATHEMATICS LEARNING

The New Brunswick Mathematics Curriculum is based upon several key assumptions or beliefs about mathematics learning which have grown out of research and practice. These beliefs include:

- mathematics learning is an active and constructive process;
- learners are individuals who bring a wide range of prior knowledge and experiences, and who learn via various styles and at different rates;
- learning is most likely to occur when placed in meaningful contexts and in an environment that supports exploration, risk taking, and critical thinking and that nurtures positive attitudes and sustained effort; and
- learning is most effective when standards of expectation are made clear with on-going assessment and feedback.

Students are curious, active learners with individual interests, abilities and needs. They come to classrooms with varying knowledge, life experiences and backgrounds. A key component in successfully developing numeracy is making connections to these backgrounds and aspirations.

Students construct their understanding of mathematics by developing meaning based on a variety of learning experiences. This meaning is best developed when learners encounter mathematical experiences that proceed from simple to complex and from the concrete to the abstract. The use of manipulatives, visuals and a variety of pedagogical approaches can address the diversity of learning styles and developmental stages of students. At all levels of understanding students benefit from working with a variety of materials, tools and contexts when constructing meaning about new mathematical ideas. Meaningful student discussions also provide essential links among concrete, pictorial and symbolic representations of mathematics. The learning environment should value, respect and address all students' experiences and ways of thinking, so that students are comfortable taking intellectual risks, asking questions and posing conjectures. Students need to explore mathematics through solving problems in order to continue developing personal strategies and mathematical literacy. It is important to realize that it is acceptable to solve problems in different ways and that solutions may vary depending upon how the problem is understood.

Goals for Mathematically Literate Students

The main goals of mathematics education are to prepare students to:

- use mathematics confidently to solve problems
- communicate and reason mathematically
- appreciate and value mathematics
- make connections between mathematics and its applications
- commit themselves to lifelong learning
- become mathematically literate adults, using mathematics to contribute to society.

Students who have met these goals will:

- gain understanding and appreciation of the contributions of mathematics as a science, philosophy and art
- exhibit a positive attitude toward mathematics
- engage and persevere in mathematical tasks and projects
- contribute to mathematical discussions

- take risks in performing mathematical tasks
- exhibit curiosity

In order to assist students in attaining these goals, teachers are encouraged to develop a classroom atmosphere that fosters conceptual understanding through:

- taking risks
- thinking and reflecting independently
- sharing and communicating mathematical understanding
- solving problems in individual and group projects
- pursuing greater understanding of mathematics
- appreciating the value of mathematics throughout history.

Opportunities for Success

A positive attitude has a profound effect on learning. Environments that create a sense of belonging, encourage risk taking, and provide opportunities for success help develop and maintain positive attitudes and self-confidence. Students with positive attitudes toward learning mathematics are likely to be motivated and prepared to learn, participate willingly in classroom activities, persist in challenging situations and engage in reflective practices.

Teachers, students and parents need to recognize the relationship between the affective and cognitive domains, and attempt to nurture those aspects of the affective domain that contribute to positive attitudes. To experience success, students must be taught to set achievable goals and assess themselves as they work toward these goals.

Striving toward success, and becoming autonomous and responsible learners are ongoing, reflective processes that involve revisiting the setting and assessing of personal goals.

Diverse Cultural Perspectives

Students come from a diversity of cultures, have a diversity of experiences and attend schools in a variety of settings including urban, rural and isolated communities. To address the diversity of knowledge, cultures, communication styles, skills, attitudes, experiences and learning styles of students, a variety of teaching and assessment strategies are required in the classroom. These strategies must go beyond the incidental inclusion of topics and objects unique to a particular culture.

For many First Nations students, studies have shown a more holistic worldview of the environment in which they live (Banks and Banks 1993). This means that students look for connections and learn best when mathematics is contextualized and not taught as discrete components. Traditionally in Indigenous culture, learning takes place through active participation and little emphasis is placed on the written word. Oral communication along with practical applications and experiences are important to student learning and understanding. It is important that teachers understand and respond to both verbal and non-verbal cues to optimize student learning and mathematical understandings.

Instructional strategies appropriate for a given cultural or other group may not apply to all students from that group, and may apply to students beyond that group. Teaching for diversity will support higher achievement in mathematics for all students.

Adapting to the Needs of All Learners

Teachers must adapt instruction to accommodate differences in student development as they enter school and as they progress, but they must also avoid gender and cultural biases. Ideally, every student should find his/her learning opportunities maximized in the mathematics classroom. The reality of individual student differences must not be ignored when making instructional decisions.

As well, teachers must understand and design instruction to accommodate differences in student learning styles. Different instructional modes are clearly appropriate, for example, for those students who are primarily visual learners versus those who learn best by doing. Designing classroom activities to support a variety of learning styles must also be reflected in assessment strategies.

Universal Design for Learning

The New Brunswick Department of Education and Early Childhood Development's definition of inclusion states that every child has the right to expect that his or her learning outcomes, instruction, assessment, interventions, accommodations, modifications, supports, adaptations, additional resources and learning environment will be designed to respect his or her learning style, needs and strengths.

Universal Design for Learning is a "…framework for guiding educational practice that provides flexibility in the ways information is presented, in the ways students respond or demonstrate knowledge and skills, and in the ways students are engaged." It also "...reduces barriers in instruction, provides appropriate accommodations, supports, and challenges, and maintains high achievement expectations for all students, including students with disabilities and students who are limited English proficient" (CAST, 2011).

In an effort to build on the established practice of differentiation in education, the Department of Education and Early Childhood Development supports *Universal Design for Learning* for all students. New Brunswick curricula are created with universal design for learning principles in mind. Outcomes are written so that students may access and represent their learning in a variety of ways, through a variety of modes. Three tenets of universal design inform the design of this curriculum. Teachers are encouraged to follow these principles as they plan and evaluate learning experiences for their students:

- **Multiple means of representation:** provide diverse learners options for acquiring information and knowledge
- **Multiple means of action and expression:** provide learners options for demonstrating what they know
- **Multiple means of engagement:** tap into learners' interests, offer appropriate challenges, and increase motivation

For further information on *Universal Design for Learning*, view online information at http://www.cast.org/.

Connections across the Curriculum

The teacher should take advantage of the various opportunities available to integrate mathematics and other subjects. This integration not only serves to show students how mathematics is used in daily life, but it helps strengthen the students' understanding of mathematical concepts and provides them with opportunities to practise mathematical skills. There are many possibilities for integrating mathematics in literacy, science, social studies, music, art, and physical education.

NATURE OF MATHEMATICS

Mathematics is one way of trying to understand, interpret and describe our world. There are a number of components that define the nature of mathematics and these are woven throughout this document. These components include: **change**, **constancy**, **number sense**, **relationships**, **patterns**, **spatial sense** and **uncertainty**.

Change

It is important for students to understand that mathematics is dynamic and not static. As a result, recognizing change is a key component in understanding and developing mathematics. Within mathematics, students encounter conditions of change and are required to search for explanations of that change. To make predictions, students need to describe and quantify their observations, look for patterns, and describe those quantities that remain fixed and those that change. For example, the sequence 4, 6, 8, 10, 12, … can be described as:

- skip counting by 2s, starting from 4
- an arithmetic sequence, with first term 4 and a common difference of 2
- a linear function with a discrete domain

(Steen, 1990, p. 184).

Students need to learn that new concepts of mathematics as well as changes to already learned concepts arise from a need to describe and understand something new. Integers, decimals, fractions, irrational numbers and complex numbers emerge as students engage in exploring new situations that cannot be effectively described or analyzed using whole numbers.

Students best experience change to their understanding of mathematical concepts as a result of mathematical play.

Constancy

Different aspects of constancy are described by the terms stability, conservation, equilibrium, steady state and symmetry (AAAS–Benchmarks, 1993, p. 270). Many important properties in mathematics and science relate to properties that do not change when outside conditions change. Examples of constancy include:

- the area of a rectangular region is the same regardless of the methods used to determine the solution
- the sum of the interior angles of any triangle is 180°
- the theoretical probability of flipping a coin and getting heads is 0.5.

Some problems in mathematics require students to focus on properties that remain constant. The recognition of constancy enables students to solve problems involving constant rates of change, lines with constant slope, direct variation situations or the angle sums of polygons.

Many important properties in mathematics do not change when conditions change. Examples of constancy include:
- the conservation of equality in solving equations
- the sum of the interior angles of any triangle
- the theoretical probability of an event.

Some problems in mathematics require students to focus on properties that remain constant. The recognition of constancy enables students to solve problems such as those involving constant rates of change, lines with constant slope, or direct variation situations.

Number Sense

Number sense, which can be thought of as deep understanding and flexibility with numbers, is the most important foundation of numeracy (British Columbia Ministry of Education, 2000, p. 146). Continuing to foster number sense is fundamental to growth of mathematical understanding.

A true sense of number goes well beyond the skills of simply counting, memorizing facts and the situational rote use of algorithms. Students with strong number sense are able to judge the reasonableness of a solution, describe relationships between different types of numbers, compare quantities and work with different representations of the same number to develop a deeper conceptual understanding of mathematics.

Number sense develops when students connect numbers to real-life experiences and when students use benchmarks and referents. This results in students who are computationally fluent and flexible with numbers and who have intuition about numbers. Evolving number sense typically comes as a by-product of learning rather than through direct instruction. However, number sense can be developed by providing mathematically rich tasks that allow students to make connections.

Patterns

Mathematics is about recognizing, describing and working with numerical and non-numerical patterns. Patterns exist in all of the mathematical topics, and it is through the study of patterns that students can make strong connections between concepts in the same and different topics.

Working with patterns also enables students to make connections beyond mathematics. The ability to analyze patterns contributes to how students understand their environment. Patterns may be represented in concrete, visual, auditory or symbolic form. Students should develop fluency in moving from one representation to another.

Students need to learn to recognize, extend, create and apply mathematical patterns. This understanding of patterns allows students to make predictions and justify their reasoning when solving problems. Learning to work with patterns helps develop students' algebraic thinking, which is foundational for working with more abstract mathematics.

Relationships

Mathematics is used to describe and explain relationships. Within the study of mathematics, students look for relationships among numbers, sets, shapes, objects, variables and concepts. The search for possible relationships involves collecting and analyzing data, analyzing patterns and describing possible relationships visually, symbolically, orally or in written form.

Spatial Sense

Spatial sense involves the representation and manipulation of 3-D objects and 2-D shapes. It enables students to reason and interpret among 3-D and 2-D representations.

Spatial sense is developed through a variety of experiences with visual and concrete models. It offers a way to interpret and reflect on the physical environment and its 3-D or 2-D representations.

Some problems involve attaching numerals and appropriate units (measurement) to dimensions of objects. Spatial sense allows students to make predictions about the results of changing these dimensions.

Spatial sense is also critical in students' understanding of the relationship between the equations and graphs of functions and, ultimately, in understanding how both equations and graphs can be used to represent physical situations

Uncertainty

In mathematics, interpretations of data and the predictions made from data may lack certainty.

Events and experiments generate statistical data that can be used to make predictions. It is important to recognize that these predictions (interpolations and extrapolations) are based upon patterns that have a degree of uncertainty. The quality of the interpretation is directly related to the quality of the data. An awareness of uncertainty allows students to assess the reliability of data and data interpretation.

Chance addresses the predictability of the occurrence of an outcome. As students develop their understanding of probability, the language of mathematics becomes more specific and describes the degree of uncertainty more accurately. This language must be used effectively and correctly to convey valuable messages.

ASSESSMENT

Ongoing, interactive assessment (*formative assessment*) is essential to effective teaching and learning. Research has shown that formative assessment practices produce significant and often substantial learning gains, close achievement gaps and build students' ability to learn new skills (Black & William, 1998, OECD, 2006). Student involvement in assessment promotes learning. Interactive assessment, and encouraging self-assessment, allows students to reflect on and articulate their understanding of mathematical concepts and ideas.

Assessment in the classroom includes:

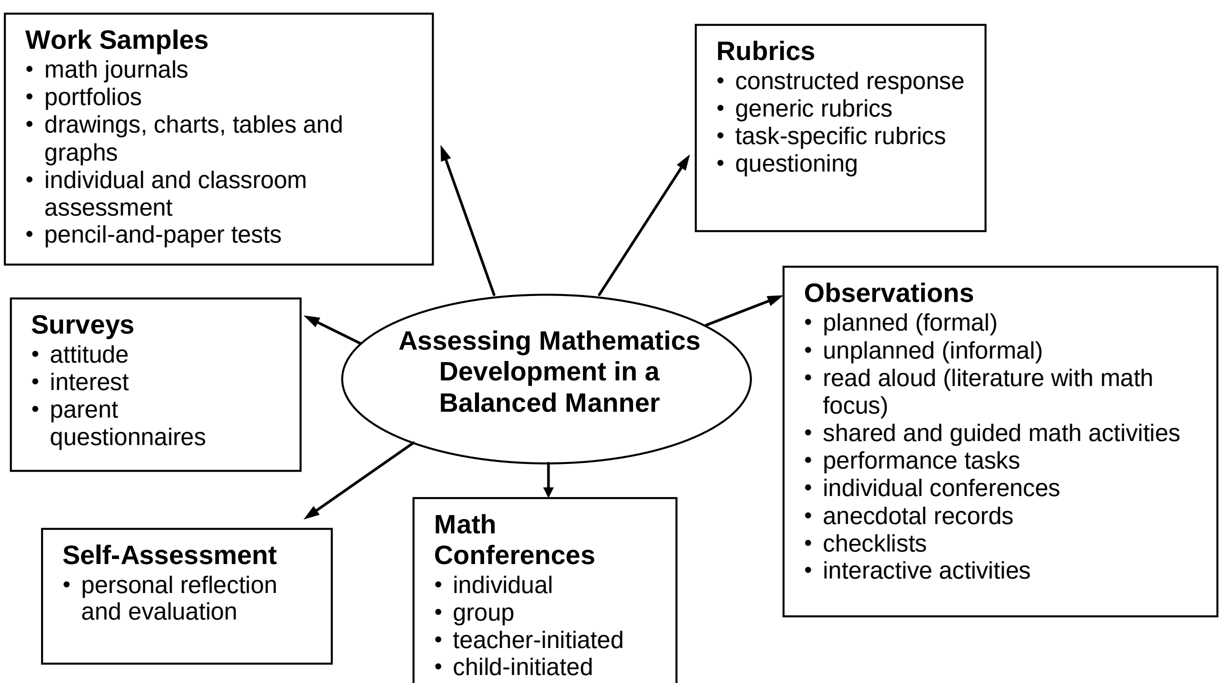
- providing clear goals, targets and learning outcomes
- using exemplars, rubrics and models to help clarify outcomes and identify important features of the work
- monitoring progress towards outcomes and providing feedback as necessary
- encouraging self-assessment
- fostering a classroom environment where conversations about learning take place, where students can check their thinking and performance and develop a deeper understanding of their learning (Davies, 2000)

Formative assessment practices act as the scaffolding for learning which, only then, can be measured through summative assessment. *Summative assessment,* or assessment *of* learning, tracks student progress, informs instructional programming and aids in decision making. Both forms of assessment are necessary to guide teaching, stimulate learning and produce achievement gains.

Student assessment should:

- align with curriculum outcomes
- use clear and helpful criteria
- promote student involvement in learning mathematics during and after the assessment experience
- use a wide variety of assessment strategies and tools
- yield useful information to inform instruction

(adapted from: NCTM, Mathematics Assessment: A practical handbook, 2001, p.22)



CONCEPTUAL FRAMEWORK FOR 10-12 MATHEMATICS

The chart below provides an overview of how mathematical processes and the nature of mathematics influence learning outcomes.

GRADE TOPICS	10 11 12
The topics of study vary in the courses for grades 10–12 mathematics. Topics in the pathways include: • *Algebra* • *Financial Mathematics* • *Geometry* • *Logical Reasoning* • *Mathematics Research Project* • *Measurement* • *Number* • *Permutations, Combinations and Binomial Theorem* • *Probability* • *Relations and Functions* • *Statistics* • *Trigonometry*	**GENERAL OUTCOMES** **SPECIFIC OUTCOMES** **ACHIEVEMENT INDICATORS**

NATURE OF MATHEMATICS

Change
Constancy
Number Sense
Patterns
Relationships
Spatial Sense
Uncertainty

MATHEMATICAL PROCESSES:
Communication, Connections, Mental Mathematics and Estimation, Problem Solving, Reasoning, Technology, Visualization

MATHEMATICAL PROCESSES

There are critical components that students must encounter in a mathematics program in order to achieve the goals of mathematics education and encourage lifelong learning in mathematics.
Students are expected to:

- communicate in order to learn and express their understanding of mathematics (Communications: C)
- connect mathematical ideas to other concepts in mathematics, to everyday experiences and to other disciplines (Connections: CN)
- demonstrate fluency with mental mathematics and estimation (Mental Mathematics and Estimation: ME)
- develop and apply new mathematical knowledge through problem solving (Problem Solving: PS)
- develop mathematical reasoning (Reasoning: R)
- select and use technologies as tools for learning and solving problems (Technology: T)
- develop visualization skills to assist in processing information, making connections and solving problems (Visualization: V).

The New Brunswick Curriculum incorporates these seven interrelated mathematical processes that are intended to permeate teaching and learning.

Communication [C]

Students need opportunities to read about, represent, view, write about, listen to and discuss mathematical ideas. These opportunities allow students to create links between their own language and ideas, and the formal language and symbols of mathematics.

Communication is important in clarifying, reinforcing and modifying ideas, knowledge, attitudes and beliefs about mathematics. Students should be encouraged to use a variety of forms of communication while learning mathematics. Students also need to communicate their learning using mathematical terminology.

Communication can help students make connections among concrete, pictorial, symbolic, verbal, written and mental representations of mathematical ideas.

Emerging technologies enable students to engage in communication beyond the traditional classroom to gather data and share mathematical ideas.

Problem Solving [PS]

Problem solving is one of the key processes and foundations within the field of mathematics. Learning through problem solving should be the focus of mathematics at all grade levels. Students develop a true understanding of mathematical concepts and procedures when they solve problems in meaningful contexts. Problem solving is to be employed throughout all of mathematics and should be embedded throughout all the topics.

When students encounter new situations and respond to questions of the type, *How would you...?* or *How could you ...?,* the problem-solving approach is being modelled. Students develop their own problem-solving strategies by listening to, discussing and trying different strategies.

In order for an activity to be problem-solving based, it must ask students to determine a way to get from what is known to what is sought. If students have already been given ways to solve the problem, it is not a problem, but practice. Students should not know the answer immediately. A true problem requires students to use prior learnings in new ways and contexts. Problem solving requires and builds depth of conceptual understanding and student engagement. Students will be engaged if the problems relate to their lives, cultures, interests, families or current events.

Both conceptual understanding and student engagement are fundamental in moulding students’ willingness to persevere in future problem-solving tasks. Problems are not just simple computations embedded in a story, nor are they contrived. They are tasks that are rich and open-ended, so there may be more than one way of arriving at a solution or there may be multiple answers. Good problems should allow for every student in the class to demonstrate their knowledge, skill or understanding. Problem solving can vary from being an individual activity to a class (or beyond) undertaking.

In a mathematics class, there are two distinct types of problem solving: solving contextual problems outside of mathematics and solving mathematical problems. Finding the maximum profit given manufacturing constraints is an example of a contextual problem, while seeking and developing a general formula to solve a quadratic equation is an example of a mathematical problem.

Problem solving can also be considered in terms of engaging students in both inductive and deductive reasoning strategies. As students make sense of the problem, they will be creating conjectures and looking for patterns that they may be able to generalize. This part of the problem-solving process often involves inductive reasoning. As students use approaches to solving the problem they often move into mathematical reasoning that is deductive in nature. It is crucial that students be encouraged to engage in both types of reasoning and be given the opportunity to consider the approaches and strategies used by others in solving similar problems.

Problem solving is a powerful teaching tool that fosters multiple, creative and innovative solutions. Creating an environment where students openly look for, and engage in, finding a variety of strategies for solving problems empowers students to explore alternatives and develops confident, cognitive mathematical risk-takers.

Connections [CN]

Contextualization and making connections to the experiences of learners are powerful processes in developing mathematical understanding. When mathematical ideas are connected to each other or to real-world phenomena, students can begin to view mathematics as useful, relevant and integrated.

Learning mathematics within contexts and making connections relevant to learners can validate past experiences, and increase student willingness to participate and be actively engaged. The brain is constantly looking for and making connections.

"*Because the learner is constantly searching for connections on many levels, educators need to orchestrate the experiences from which learners extract understanding… Brain research establishes and confirms that multiple complex and concrete experiences are essential for meaningful learning and teaching*" (Caine and Caine, 1991, p. 5).

Mental Mathematics and Estimation [ME]

Mental mathematics is a combination of cognitive strategies that enhance flexible thinking and number sense. It is calculating mentally without the use of external memory aids.

Mental mathematics enables students to determine answers without paper and pencil. It improves computational fluency by developing efficiency, accuracy and flexibility.

"Even more important than performing computational procedures or using calculators is the greater facility that students need—more than ever before—with estimation and mental mathematics" (National Council of Teachers of Mathematics, May 2005).

Students proficient with mental mathematics "*become liberated from calculator dependence, build confidence in doing mathematics, become more flexible thinkers and are more able to use multiple approaches to problem solving*" (Rubenstein, 2001).

Mental mathematics "*provides a cornerstone for all estimation processes offering a variety of alternate algorithms and non-standard techniques for finding answers*" (Hope, 1988).

Estimation is a strategy for determining approximate values or quantities, usually by referring to benchmarks or using referents, or for determining the reasonableness of calculated values. Students need to know how, when and what strategy to use when estimating. Estimation is used to make mathematical judgments and develop useful, efficient strategies for dealing with situations in daily life. When estimating, students need to learn which strategy to use and how to use it.

Technology [T]

Technology can be used effectively to contribute to and support the learning of a wide range of mathematical outcomes. Technology enables students to explore and create patterns, examine relationships, test conjectures and solve problems.

Calculators and computers can be used to:
• explore and demonstrate mathematical relationships and patterns
• organize and display data
• generate and test inductive conjectures
• extrapolate and interpolate
• assist with calculation procedures as part of solving problems
• increase the focus on conceptual understanding by decreasing the time spent on repetitive procedures
• reinforce the learning of basic facts
• develop personal procedures for mathematical operations
• model situations
• develop number and spatial sense.

Technology contributes to a learning environment in which the curiosity of students can lead to rich mathematical discoveries at all grade levels. The use of technology should not replace mathematical understanding. Instead, technology should be used as one of a variety of approaches and tools for creating mathematical understanding.

Visualization [V]

Visualization "involves thinking in pictures and images, and the ability to perceive, transform and recreate different aspects of the visual-spatial world" (Armstrong, 1993, p. 10). The use of visualization in the study of mathematics provides students with opportunities to understand mathematical concepts and make connections among them.

Visual images and visual reasoning are important components of number, spatial and measurement sense. Number visualization occurs when students create mental representations of numbers. Being able to create, interpret and describe a visual representation is part of spatial sense and spatial reasoning. Spatial visualization and spatial reasoning enable students to describe the relationships among and between 3-D objects and 2-D shapes.

Measurement visualization goes beyond the acquisition of specific measurement skills. Measurement sense includes the ability to determine when to measure and when to estimate and involves knowledge of several estimation strategies (Shaw and Cliatt, 1989, p. 150).

Visualization is fostered through the use of concrete materials, technology and a variety of visual representations. It is through visualization that abstract concepts can be understood concretely by the student. Visualization is a foundation to the development of abstract understanding, confidence and fluency.

Reasoning [R]

Mathematical reasoning helps students think logically and make sense of mathematics. Students need to develop confidence in their abilities to reason and justify their mathematical thinking.

Questions that challenge students to think, analyze and synthesize help them develop an understanding of mathematics. All students need to be challenged to answer questions such as, *Why do you believe that's true/correct?* or *What would happen if ….*

Mathematical experiences provide opportunities for students to engage in inductive and deductive reasoning. Students use inductive reasoning when they explore and record results, analyze observations, make generalizations from patterns and test these generalizations. Students use deductive reasoning when they reach new conclusions based upon the application of what is already known or assumed to be true. The thinking skills developed by focusing on reasoning can be used in daily life in a wide variety of contexts and disciplines.

ESSENTIAL GRADUATION LEARNINGS

Graduates from the public schools of Atlantic Canada will be able to demonstrate knowledge, skills, and attitudes in the following essential graduation learnings. These learnings are supported through the outcomes described in this curriculum document.

Aesthetic Expression
Graduates will be able to respond with critical awareness to various forms of the arts and be able to express themselves through the arts.

Citizenship
Graduates will be able to assess social, cultural, economic, and environmental interdependence in a local and global context.

Communication
Graduates will be able to use the listening, viewing, speaking, reading and writing modes of language(s) as well as mathematical and scientific concepts and symbols to think, learn, and communicate effectively.

Personal Development
Graduates will be able to continue to learn and to pursue an active, healthy lifestyle.

Problem Solving
Graduates will be able to use the strategies and processes needed to solve a wide variety of problems, including those requiring language, mathematical, and scientific concepts.

Technological Competence
Graduates will be able to use a variety of technologies, demonstrate an understanding of technological applications, and apply appropriate technologies for solving problems

PATHWAYS AND TOPICS

The Common Curriculum Framework for Grades 10–12 Mathematics on which the New Brunswick Grades 10-12 Mathematics curriculum is based, includes pathways and topics rather than strands as in *The Common Curriculum Framework for K–9 Mathematics*. In New Brunswick all Grade 10 students share a common curriculum covered in two courses: *Geometry, Measurement and Finance 10* and *Number, Relations and Functions 10*. Starting in Grade 11, three pathways are available: *Finance and Workplace*, *Foundations of Mathematics*, and *Pre-Calculus*.

Each topic area requires that students develop a conceptual knowledge base and skill set that will be useful to whatever pathway they have chosen. Students are encouraged to cross pathways to follow their interests and to keep their options open. The topics covered within a pathway are meant to build upon previous knowledge and to progress from simple to more complex conceptual understandings.

Goals of Pathways

The goals of all three pathways are to provide prerequisite attitudes, knowledge, skills and understandings for specific post-secondary programs or direct entry into the work force. All three pathways provide students with mathematical understandings and critical-thinking skills. It is the choice of topics through which those understandings and skills are developed that varies among pathways. When choosing a pathway, students should consider their interests, both current and future. Students, parents and educators are encouraged to research the admission requirements for post-secondary programs of study as they vary by institution and by year.

Design of Pathways

Each pathway is designed to provide students with the mathematical understandings, rigour and critical-thinking skills that have been identified for specific post-secondary programs of study and for direct entry into the work force.

The content of each pathway has been based on the *Western and Northern Canadian Protocol (WNCP) Consultation with Post-Secondary Institutions, Business and Industry Regarding Their Requirements for High School Mathematics: Final Report on Findings* and on consultations with mathematics teachers.

<u>Financial and Workplace Mathematics</u>

This pathway is designed to provide students with the mathematical understandings and critical-thinking skills identified for entry into some college programs and for direct entry into the work force. Topics include financial mathematics, algebra, geometry, measurement, number, statistics and probability.

<u>Foundations of Mathematics</u>

This pathway is designed to provide students with the mathematical understandings and critical-thinking skills identified for post-secondary studies in programs that do not require the study of theoretical calculus. Topics include financial mathematics, geometry, measurement, number, logical reasoning, relations and functions, statistics and probability.

Pre-calculus

This pathway is designed to provide students with the mathematical understandings and critical-thinking skills identified for entry into post-secondary programs that require the study of theoretical calculus. Topics include algebra and number, measurement, relations and functions, trigonometry, and permutations, combinations and binomial theorem.

Outcomes and Achievement Indicators

The New Brunswick Curriculum is stated in terms of general curriculum outcomes, specific curriculum outcomes and achievement indicators.

General Curriculum Outcomes (GCO) are overarching statements about what students are expected to learn in each strand/sub-strand. The general curriculum outcome for each strand/sub-strand is the same throughout the pathway.

Specific Curriculum Outcomes (SCO) are statements that identify specific concepts and related skills underpinned by the understanding and knowledge attained by students as required for a given grade.

Achievement indicators are samples of how students may demonstrate their achievement of the goals of a specific outcome. The range of samples provided is meant to reflect the scope of the specific outcome. In the specific outcomes, the word *including* indicates that any ensuing items must be addressed to fully meet the learning outcome. The phrase *such as* indicates that the ensuing items are provided for clarification and are not requirements that must be addressed to fully meet the learning outcome. The word *and* used in an outcome indicates that both ideas must be addressed to fully meet the learning outcome, although not necessarily at the same time or in the same question.

Instructional Focus

Each pathway in *The Common Curriculum Framework for Grades 10–12 Mathematics* is arranged by topics. Students should be engaged in making connections among concepts both within and across topics to make mathematical learning experiences meaningful. Teachers should consider the following points when planning for instruction and assessment.

- The mathematical processes that are identified with the outcome are intended to help teachers select effective pedagogical approaches for the teaching and learning of the outcome.
- All seven mathematical processes must be integrated throughout teaching and learning approaches, and should support the intent of the outcomes.
- Wherever possible, meaningful contexts should be used in examples, problems and projects.
- Instruction should flow from simple to complex and from concrete to abstract.
- The assessment plan for the course should be a balance of assessment for learning, assessment as learning and assessment of learning.

The focus of student learning should be on developing a conceptual and procedural understanding of mathematics. Students' conceptual understanding and procedural understanding must be directly related.

Pathways and Courses

The graphic below summarizes the pathways and courses offered.

Mathematics K-9

Grade 10

- *2 x 90 hr courses; required to pass both*
- *May be taken in any order or in the same semester*

Geometry, Measurement and Finance 10 (1069027)

Number, Relations and Functions 10 (1069527)

Grade 11

- *3 x 90 hr courses offered in 3 pathways*
- *Students are required to pass at least one of "Financial and Workplace Mathematics 11" **or** "Foundations of Mathematics 11".*
- *Pre-requisite Grade 10 course(s) must be passed before taking Grade 11 courses.*

Financial and Workplace Mathematics 11	Foundations of Mathematics 11	Pre-Calculus 11
Pre-requisite: Geometry, Measurement and Finance 10	*Pre-requisites: Geometry, Measurement and Finance 10 AND Number, Relations and Functions 10*	*Pre-requisite or Co-requisite: Foundations of Mathematics 11*

Grade 12

- *5 x 90 hr courses offered in 3 pathways*
- *Pre-requisite Grade 11 or Grade 12 course must be passed before taking Grade 12 courses.*

Financial and Workplace Mathematics 12	Foundations of Mathematics 12	Pre-Calculus 12A
Pre-requisite: Financial and Workplace Mathematics 11	*Pre-requisite: Foundations of Mathematics 11*	*Pre-requisite: Pre-Calculus 11*
		Pre-Calculus 12B
		Pre-requisite: Pre-Calculus 12A
		Calculus 12
		Pre-requisite: Pre-Calculus 12B

SUMMARY

The Conceptual Framework for Grades 10–12 Mathematics describes the nature of mathematics, the mathematical processes, the pathways and topics, and the role of outcomes and achievement indicators in grades 10–12 mathematics. Activities that take place in the mathematics classroom should be based on a problem-solving approach that incorporates the mathematical processes and leads students to an understanding of the nature of mathematics.

CURRICULUM DOCUMENT FORMAT

This guide presents the mathematics curriculum by grade level so that a teacher may readily view the scope of the outcomes which students are expected to meet during that year. Teachers are encouraged, however, to examine what comes before and what follows after, to better understand how the students' learnings at a particular grade level are part of a bigger picture of concept and skill development.

The order of presentation in no way assumes or prescribes a preferred order of presentation in the classroom, but simply lays out the specific curriculum outcomes in relation to the overarching general curriculum outcomes (GCOs).

The heading of each page gives the General Curriculum Outcome (GCO), and Specific Curriculum Outcome (SCO). The key for the mathematical processes follows. A <u>Scope and Sequence</u> is then provided which relates the SCO to previous and next grade SCO's. For each SCO, <u>Elaboration</u>, <u>Achievement Indicators</u>, <u>Suggested Instructional Strategies</u>, and <u>Suggested Activities for Instruction and Assessment</u> are provided. For each section, the *Guiding Questions* should be considered.

GCO: General Curriculum Outcome
SCO: Specific Curriculum Outcome

Mathematical Processes

[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Math and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

<u>Scope and Sequence</u>

Previous Grade or Course SCO's	Current Grade SCO	Following Grade or Course SCO's

<u>Elaboration</u>

Describes the "big ideas" to be learned and how they relate to work in previous Grades

Guiding Questions:
- *What do I want my students to learn?*
- *What do I want my students to understand and be able to do?*

<u>Achievement Indicators</u>

Describes observable indicators of whether students have met the specific outcome

<u>Guiding Questions:</u>
- *What evidence will I look for to know that learning has occurred?*
- *What should students demonstrate to show their understanding of the mathematical concepts and skills?*

GCO: General Curriculum Outcome
SCO: Specific Curriculum Outcome

<u>Suggested Instructional Strategies</u>

General approach and strategies suggested for teaching this outcome

<u>Guiding Questions</u>
- *What learning opportunities and experiences should I provide to promote learning of the outcomes and permit students to demonstrate their learning?*
- *What teaching strategies and resources should I use?*
- *How will I meet the diverse learning needs of my students?*

<u>Suggested Activities for Instruction and Assessment</u>

Some suggestions of specific activities and questions that can be used for both instruction and assessment.

<u>Guiding Questions</u>
- *What are the most appropriate methods and activities for assessing student learning?*
- *How will I align my assessment strategies with my teaching strategies?*

<u>Guiding Questions</u>
- *What conclusions can be made from assessment information?*
- *How effective have instructional approaches been?*
- *What are the next steps in instruction?*

Geometry, Measurement and Finance 10

Specific Curriculum Outcomes

SCO	**A1: Solve problems that require the manipulation and application of formulas related to: perimeter, area, volume, capacity, the Pythagorean theorem, primary trigonometric ratios, income, currency exchange, interest and finance charges.** [CN, ME, PS, R]

Algebra

[C] Communication **[T]** Technology	**[PS]** Problem Solving **[V]** Visualization	**[CN]** Connections **[R]** Reasoning	**[ME]** Mental Math and Estimation

Scope and Sequence of Outcomes

Grade Nine	Grade Ten	Grade Eleven
PR3: Model and solve problems using linear equations of the form: $ax = b$; $\frac{x}{a} = b$, $a \neq 0$; $ax + b = c$; $\frac{x}{a} + b = c$, $a \neq 0$; $ax = b + cx$; $a(x + b) = c$; $ax + b = cx + d$; $a(bx + c) = d(ex + f)$; $\frac{a}{x} = b$, $x \neq 0$ where *a*, *b, c, d, e* and *f* are rational numbers. **SS2:** Determine the surface area of composite 3-D objects to solve problems.	**A1**: Solve problems that require the manipulation and application of formulas related to: perimeter, area, volume, capacity, the Pythagorean theorem, primary trigonometric ratios, income. currency exchange, interest and finance charges.	**A1**: Solve problems that require the manipulation and application of formulas, related to slope and rate of change *(FWM11)* **N1:** Analyze costs and benefits of renting, leasing and buying. *(FM11)* **G3:** Solve problems that involve the cosine law and the sine law, including the ambiguous case. *(FM11)*

ELABORATION

Students have been modeling and solving various forms of linear equations since Grade 7 and have practiced manipulating these equations to solve for an unknown variable.

This skill should be further developed and the outcome addressed **throughout this course** as students apply formulas in a variety of contexts such as income, currency exchange, perimeter, area, volume, capacity, the Pythagorean theorem, and trigonometric ratios.

This is not an outcome that can be taught in isolation but must be integrated as a foundational concept within each of the units in the course.

ACHIEVEMENT INDICATORS

- Solve a contextual problem that involves the application of a formula that does not require manipulation.
- Solve a contextual problem that involves the application of a formula that requires manipulation.
- Explain and verify why different forms of the same formula are equivalent.
- Create and solve a contextual problem that involves a formula.
- Identify and correct errors in a solution to a problem that involves a formula.

SCO	**A1: Solve problems that require the manipulation and application of formulas related to: perimeter, area, volume, capacity, the Pythagorean theorem, primary trigonometric ratios, income, currency exchange, interest and finance charges.** [CN, ME, PS, R]

Suggested Instructional Strategies

- When teaching each topic in this course, ensure that students can apply and manipulate the appropriate formulas.
- following topics: currency exchange, unit pricing, proportional reasoning, wages and salaries, credit and loans, the Pythagorean theorem, trigonometric ratios, parallel, perpendicular and transversal lines.

Suggested Questions (Q) and Activities (Act) for Instruction and Assessment

This outcome is a foundational concept throughout this course, and so should be referred back to frequently as the course proceeds.

To address this outcome, examples of formula use can be found within the document in the “Suggested Instructional Strategies” and the “Suggested Questions (Q) and Activities (Act) for Instruction and Assessment”.

SCO	**N1: Solve problems that involve unit pricing and currency exchange, using proportional reasoning.** [C,CN, ME, PS, R]

Number

[C] Communication **[T]** Technology	**[PS]** Problem Solving **[V]** Visualization	**[CN]** Connections **[R]** Reasoning	**[ME]** Mental Math and Estimation

Scope and Sequence of Outcomes

Grade Nine	Grade Ten	Grade Eleven
N3 Demonstrate an understanding of rational numbers by comparing and ordering rational numbers, and solving problems that involve arithmetic operations on rational numbers. **SS4** Draw and interpret scale diagrams of 2-D shapes (similar triangles)	**N1:** Solve problems that involve unit pricing and currency exchange, using proportional reasoning.	**N2**: Analyze costs and benefits of renting, leasing and buying. *(FWM11)* **N3**: Analyze an investment portfolio in terms of interest rate, rate of return, total return. *(FWM11)* **A3:** Solve problems by applying proportional reasoning and unit analysis. *(FWM11)* **N1**: Analyze costs and benefits of renting, leasing and buying. *(FM11)* **N2**: Analyze an investment portfolio in terms of interest rate, rate of return, total return. *(FM11)* **PR1:** Solve problems that involve the application of rates *(FM11)* **PR2:** Solve problems that involve scale diagrams, using proportional reasoning.*(FM11)*

ELABORATION

In Grade 9 students were introduced to proportional reasoning through their study of similar triangles. Students will need to develop proficiency in this skill in order to extend it to everyday situations such as shopping, calculating taxes and currency exchange. Teachers should work from simpler to more complex examples as students' proficiency increases.

Proportional reasoning will be used in estimating and calculating the **unit price**. Estimation and proportional reasoning are skills that have been identified as weaknesses in our adult population, but are critical components of financial literacy.

For students to become financially literate consumers, they must be able to estimate and/or calculate total cost, taking into account discounts and additional costs such as taxes and shipping. The students will also take into account other factors such as ethics, product quality and practicality before making a purchase.

On a more global level, this topic will allow students to explore the use of ratio to estimate or calculate a currency value based on fluctuating currency rates.

SCO	**N1: Solve problems that involve unit pricing and currency exchange, using proportional reasoning.** [CN, ME, PS, R]

ACHIEVEMENT INDICATORS

- Compare the unit price of two or more given items.
- Solve problems that involve determining the best buy, and explain the choice in terms of the cost as well as other factors, such as quality and quantity.
- Compare, using examples, different sales promotion techniques; deli meat at $2 per 100 g seems less expensive than $20 per kilogram.
- Determine the increase or decrease of the original price based on percentage sale discounts or extra charges.
- Solve, using proportional reasoning, a contextual problem that involves currency exchange.
- Explain the difference between the selling rate and purchasing rate for currency exchange.
- Explain how to estimate the cost of items in Canadian currency while in a foreign country, and explain why this may be important.
- Convert between Canadian currency and foreign currencies, using formulas, charts or tables.

Suggested Instructional Strategies

- Activate prior knowledge of proportions and percentage seen in previous years.
- Estimation should be emphasized before a student calculates the answer, to predict whether the answer is reasonable or not.
- Develop proportional reasoning through a progression of difficulty of questions. Start with simple numbers (whole number examples that are easy to double, triple etc.) to establish the understanding before using more complex numbers and examples

  How much for one of something→ two of something→ four of something" using real life examples
- Use flyers, catalogues, and websites to provide real-life examples. Bring in products, such as yogurt, cereal, granola bars, vitamins, so students can visually make comparisons.
- Check currency rates as a class and then talk about daily and longer term fluctuations.

SCO	**N1: Solve problems that involve unit pricing and currency exchange, using proportional reasoning.** [CN, ME, PS, R]

<u>Suggested Questions (Q) and Activities (Act) for Instruction and Assessment</u>

Q These questions illustrate increasing levels of difficulty to develop proportional reasoning skills:

- John is saving $80 every 3 weeks. In how many weeks will he have $160 saved?
- It takes 45 minutes to wash 5 cars. How much time will it take to wash 10 cars?
- Jill can make 3 dozen cookies with 1 ½ cups of flour. How many cups of flour will she need to make 9 dozen cookies?
- It takes 10 balls of wool to make 15 scarves. How many balls of wool does it take to make 6 beanies?
- I travelled 40 km in 80 minutes. How far did I travel in an hour?
- Four grey garden tiles are used for every three red ones. If you use 210 tiles determine how many are grey and how many are red?
- There are 3 oranges for every one apple in the bowl. How many oranges will there be if there are 5 apples? How many apples and oranges are there if the bowl has 20 pieces of fruit in it?
- It takes Kara 8 hours to wash 12 cars. How long will it take her to wash 21 cars?
- When mixing paint, which combination will result in the bluer shade of green - 2 parts blue with 3 parts yellow <u>or</u> 3 parts blue with 5 parts yellow?
- You can buy 8 avocadoes for $6. How many can you buy for $15?
- Peter spent two-fifths of his savings, and had $30 left. How much did he spend?
- Jeremy invited his friends to a BBQ. Twenty-four friends happily attended, but 16 had other commitments. What percentage of Jeremy's friends was at the BBQ?
- Mathieu ate $^5/_9$ of a box of chocolates. That left only 16 chocolates for his brother Michael. How many chocolates were there in the box at the start?
- 80 :120 is the same as 4:___

Q First estimate your answer, then solve for x. Compare the estimated and calculated answers to check if your answer is correct.

a) $\frac{2.68}{5\ rolls} = \frac{x}{1\ roll}$

b) $\frac{x}{300} = \frac{1}{1.56}$

c) A box of 12 pencils costs $1.69. How much does one pencil cost?

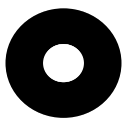
SCO **N1: Solve problems that involve unit pricing and currency exchange, using proportional reasoning.**
[CN, ME, PS, R]

Act Have student choose 5 foreign vehicles and research the following:

Country of origin	Make and Model	Name of currency	Exchange Rate	Value in Foreign currency	Value in Canadian $

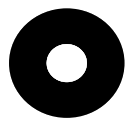
Q 4L of house paint sells for $42.95, and 250 mL for $7.50. The first option (4L) requires the purchase of primer, while the second option (250 mL) has the primer included. Which option would you have chosen and what conditions would affect your choice? *(note to teacher: this is an open question and either option has its merits – students should be given the opportunity to explore both options and the merits of each depending on the situation)*

Q *Tires for All Inc.*



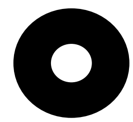
Regular $118
Now 30% off!

Best Value Tires Inc.



Regular $118
Buy one, get 2nd half-off!

Treads –R-Us



Regular $118
Buy 3, Get one free!

You need 4 new winter tires. Which company has the better deal? Discuss the promotional techniques offered by each company *(teacher note: use real examples of sales, promotions…. that are found in your community in local flyers).*

Q You have purchased an iPod touch for $225. The original price was $300. What percentage discount did you receive?

Q For a school exchange trip to Europe, your parents have given you $500 CAD as spending money. At the bank, you exchanged this for Euros, at a rate of *1 EUR = 1.35 CAD*. How many Euros will you receive? *(Ans: €370.37)* At this exchange rate, what is the value of 1 CAD in Euros? *(Ans: €0.74)*

On return you have €50 left, and you exchange these Euros back into Canadian dollars at a rate of *1EUR =1.27 CAD*. How much will you have lost (paid to the bank) for the exchange of this €50 back and forth? *(Ans: $4)*

(teacher note: actual current rates can be found online and used in place of rates given)

Q Prior to your trip to Mexico you exchanged some Canadian Dollars for Pesos at an exchange rate of *1 CAD =12.35 MXN.* You buy a burrito for $30 (30 Pesos). Approximately how much has this cost you in Canadian dollars?

Act Have students can go online to explore the daily exchange rate over the last 30 days, and how it may have been influenced by current events or other factors.

SCO	**N2: Demonstrate an understanding of income, including: wages, salary, contracts, commissions, and piecework to calculate gross pay and net pay.** [C, CN, ME, R, T]

[C] Communication **[T]** Technology	**[PS]** Problem Solving **[V]** Visualization	**[CN]** Connections **[R]** Reasoning	**[ME]** Mental Math and Estimation

Scope and Sequence of Outcomes

Grade Nine	Grade Ten	Grade Eleven
N3 Demonstrate an understanding of rational numbers by: comparing and ordering rational numbers solving problems that involve arithmetic operations on rational numbers. **N4** Explain and apply the order of operations, including exponents, with and without technology.	**N2:** Demonstrate an understanding of income, including: wages, salary, contracts, commissions, and piecework to calculate gross pay and net pay.	**N2**: Analyze costs and benefits of renting, leasing and buying. *(FWM11)* **N4**: Solve problems that involve personal budgets *(FWM11)* **N1**: Analyze costs and benefits of renting, leasing and buying. *(FM11)*

ELABORATION

Students will gain an understanding of **income**, how it can be earned and what the advantages and disadvantages of various ways of earning an income might be.

Income is the money received within a specified time frame, usually in return for work completed. This can be in the form of an **hourly wage** in which a worker is paid at a set rate per hour, a **wage** as **piecework** in which a worker is paid a fixed “piece rate” for each unit produced or job completed (e.g. planting trees, completing a translation job), or a **salary** which is paid periodically by an employer to an employee, and may be specified in an employment **contract**.

Working **on commission** involves an employee performing a service or making a sale for a business and being paid a percentage of the money received by their employer for each service performed or sale made. Workers can work entirely on commission or work for a base salary and receive a commission over and above their salary. Extra pay can also be received if working overtime or on holidays, or as tips, bonuses, or shift premiums.

Applying *SCO A1*, students will use formulae to calculate income, and **gross** and **net pay**. They will determine which **deductions** are required and which are optional depending on circumstances. They will understand that gross pay is what you make before any deductions. Net pay is the actual “take home” pay after taxes, health benefits CPP, EI and other deductions are taken into account.

SCO **N2: Demonstrate an understanding of income, including: wages, salary, contracts, commissions, and piecework to calculate gross pay and net pay.**
[C, CN, ME, R, T]

ACHIEVEMENT INDICATORS

- Describe, using examples, various methods of earning income.
- Identify and list jobs that commonly use different methods of earning income such as: hourly wage, wage and tips, piecework, salary, commission, contract, bonus, shift premiums.
- Describe the advantages and disadvantages of a given method of earning income listed above.
- Calculate, in decimal form, from a time schedule, the total time worked in hours and minutes, including time and a half and/or double time.
- Calculate gross pay from given or calculated hours worked when given: the base hourly wage with and without tips, and the base hourly wage plus overtime (time and a half, double time).
- Calculate gross pay for earnings acquired by base wage plus commission, and single commission rate
- Explain the difference between gross pay and net pay.
- Calculate the Canadian Pension Plan (CPP), Employment Insurance (EI) and Income Tax deductions for a given gross pay.
- Calculate net pay when given deductions such as health plans, uniforms, union dues, charitable donations, payroll tax.
- Identify and correct errors in a solution to a problem that involves gross or net pay.

Suggested Instructional Strategies

- Have students generate a list of jobs that interest them and then have them work in teams to categorize the jobs by payment method (salary, hourly wage, commission).

  As a class, discuss other payment methods, and the advantages and disadvantages of each method.
- Provide students with a scenario and have them complete a pay stub, filling in all the important information (EI, CPP, gross pay, net pay, etc.).
- Teachers should visit the CRA website yearly for the updated rates for Employment Insurance (EI), Canada Pension Plan (CPP), and tax tables.

SCO	**N2: Demonstrate an understanding of income, including: wages, salary, contracts, commissions, and piecework to calculate gross pay and net pay.** [C, CN, ME, R, T]

Suggested Questions (Q) and Activities (Act) for Instruction and Assessment

Q Monica's monthly gross income is $1916.67

a) Calculate her Employment Insurance (EI) payment using the formula given

b) Calculate her monthly Canada Pension Plan (CPP) payment using the formula given.

c) If other deductions total $235, calculate Monica's monthly net income.

(Note to teacher: For a) and b) check rate for EI and CPP which change yearly, and provide students with the appropriate formula)

Q Sammy is a sales clerk in a bicycle shop. He's paid $11.25/hour for a 37.5 hour week, plus a commission of 6% of his sales for the week. In one week Sammy's sales were $2319.75.

a) Calculate Sammy's gross pay for the week.

b) What was his average hourly wage for that week?

c) What would Sammy's sales for the week have to be for him to earn $700 in one week?

Act Have students research the business and classified ads sections of the newspaper. Find a job where you would be paid:

a) A salary

b) An hourly wage

c) A straight commission

d) A salary plus commission

e) An hourly wage plus commission

f) By piece work

From the figures you have acquired on your ads, calculate the yearly, monthly and weekly gross wages available for each job. Describe the advantages and disadvantages of one of the methods of earning income you have researched.

SCO **N2: Demonstrate an understanding of income, including: wages, salary, contracts, commissions, and piecework to calculate gross pay and net pay.**
[C, CN, ME, R, T]

Q Complete the following time schedule for each employee:

Employee hours

Regular Hours : 8 hrs Overtime Rate : 1.5

Name	*Start*	*Lunch Out*	*Lunch In*	*End*	*Total hours*	*Worked Hours*	*Regular Hours*	*Overtime Hours*	*$/hr*	*Reg $*	*Over $*	*Total*
Ryan	9 :00	12 :00	13 :00	18 :00	9 :00	8 :00	8 :00	0 :00	$11.00	$88.00	$0.00	$88.00
Sheila	8 :30	11 :30	12 :30	18 :30					$11.00			
Katelyn	8 :45	12 :30	13 :15	16 :30					$12.25			
Simon	22 :00	0 :30	1 :00	9 :00					$11.50			

Q Claudette has been offered two jobs, one of which offers an annual salary of $37 500, and the other which requires working a 40 hour week, 50 weeks a year for $18.25/hour.

a) Calculate the weekly pay for each option.

b) Which option gives Claudette the greatest gross income?

Q As a waitress, Carla earns $7.25/hour for a 40 hour week and shares 25% of her tips with other employees. In one week, her tips were $318. What was Sarah's gross pay for the week?

Q Pat is paid $8.50/h for a 37.5 h week and earns double-time for overtime.

a) Calculate Pat's gross pay if her total over-time was 4.5 hours.

b) Determine C.P.P., E.I., & Income Tax deduction amounts

c) Calculate net pay if the other deductions total $14.73.

Q Identify and <u>**correct the error made**</u> when solving the following problem:

A real estate agent earns 2.4% on the sale of a house. The last house she sold earned her a commission of $4128. What was the selling price of the house?

$$Commission\ (C) = Selling\ Price(P) \times Commission\ Rate\ (R)$$

$$C = PR$$

$$4128 = P \times 2.4\%$$

$$P = 4128 \times 2.4\%$$

$$P = \$9907.20$$

(note to teacher: the correct calculation is $\$4\,128 \div 2.4\% = \$172\,000$)

SCO	**N3: Demonstrate an understanding of financial institution services used to access and manage finances.** [C, CN, R, T]

[C] Communication **[T]** Technology	**[PS]** Problem Solving **[V]** Visualization	**[CN]** Connections **[R]** Reasoning	**[ME]** Mental Math and Estimation

Scope and Sequence of Outcomes

Grade Nine	Grade Ten	Grade Eleven
	N3: Demonstrate an understanding of financial institution services used to access and manage finances.	**N2**: Analyze costs and benefits of renting, leasing and buying. *(FWM11)* **N3**: Analyze an investment portfolio in terms of interest rate, rate of return, total return. *(FWM11)* **N4:** Solve problems that involve personal budgets (*FWM11)* **N1**: Analyze costs and benefits of renting, leasing and buying. *(FM11)* **N2**: Analyze an investment portfolio in terms of interest rate, rate of return, total return. *(FM11)*

ELABORATION

Students will have various levels of experiences with financial institutions. This is an opportunity to become familiar with the terminology used, to become aware of the services offered by financial institutions, and to critically consider the best options for them depending upon their situation.

The following terminology may be new to students, depending upon their life experience. **Interest** refers to money earned on an investment or a fee paid for borrowing money, usually expressed as a percentage. **Self-service banking** is banking done over the internet, by telephone, or at a banking machine, that does not require the services of a teller. **Full-service banking** is banking that is done with the help of a teller. **Transactions** include any activity involving money such as a cash withdrawal, deposit, money transfer, pre-authorized payment, or a bill payment. A **PIN** or **Personal Identification Number** is a secret numeric password that is used by a computer system to verify the identity of the user.

ACHIEVEMENT INDICATORS

- Describe the type of banking services available from various financial institutions, such as online services.
- Describe the types of accounts available at various financial institutions.
- Identify the type of account that best meets the needs for a given set of criteria.
- Identify and explain various automated teller machine (ATM) service charges.
- Describe the advantages and disadvantages of online banking.
- Describe the advantages and disadvantages of debit card purchases.
- Describe ways that ensure the security of personal and financial information; passwords, encryption, protection of personal identification number (PIN) and other personal identity information.

SCO	**N3: Demonstrate an understanding of financial institution services used to access and manage finances.** [C, CN, R, T]

Suggested Instructional Strategies

- There will likely be a wide variation of experiences with managing money among your students. Encourage those that understand and have experience with their own money, to share their knowledge and experience, though care should be taken to maintain the privacy of students and their families.
- Local financial banking institutions will have brochures and websites outlining fees for various account options. Have students compile this information, and consider their needs, and then determine which institution and which account would be best for them.
- Invite representatives from various financial institutions in your community to come to the school and explain account options and banking services to the students.

Suggested Questions (Q) and Activities (Act) for Instruction and Assessment

Q If you have your own bank or credit union account, keep a record of frequency of various types of transactions you make over the course of one or two months. Compare the account you have, with other account options and determine if you have the best account for your needs. Explain the various reasons for your choice.

Act Have different groups of students research account and fee options at various local banks and credit unions and present their findings to one another. Include information on how credit unions differ from banks.

Act Have a guest speaker from a financial institution come to your class and discuss the topics listed in the achievement indicators.

Act Provide students with a profile of a fictitious person who they have to serve as the bank employee assisting this person. Have them research one or more bank accounts that would best meet this person's needs.

Act Provide students with a graphic organizer and have them research various banks and compare services.

SCO	**N4 Demonstrate an understanding of simple and compound interest.** [CN, ME, PS, T]

[C] Communication Technology	**[PS]** Problem Solving **[V]** Visualization	**[CN]** Connections **[R]** Reasoning	**[ME]** Mental Math **[T]** and Estimation

Scope and Sequence of Outcomes

Grade Nine	Grade Ten	Grade Eleven
	N4: Demonstrate an understanding of simple and compound interest.	**N2**: Analyze costs and benefits of renting, leasing and buying. *(FWM11)* **N3**: Analyze an investment portfolio in terms of interest rate, rate of return, total return. *(FWM11)* **N1**: Analyze costs and benefits of renting, leasing and buying. *(FM11)* **N2**: Analyze an investment portfolio in terms of interest rate, rate of return, total return. *(FM11)*

ELABORATION

Beginning with calculations for simple interest, students will then practice calculating compound interest. New terminology will include:

Principal: the original amount invested or borrowed
Interest rate: the percentage charged, usually stated as a per year rate
Simple interest: interest calculated as a percentage of the principal
Term: the time in years for an investment or loan
Compound interest: the interest paid on the principal plus interest
Compounding period: the time between calculation of interest, also called the interest period

To calculate simple interest students will use the formula: ***I=Prt.***
P= the principal r = rate of interest t = term of the investment

To calculate the value of investment with compound interest over a long term students will use the following formula:

$$A = P\left(1+\frac{r}{n}\right)^{nt}$$

P = the principle r = annual interest rate
n = number of compounding periods in a year
t = term of the investment or loan in years

A method that is used to estimate the time it takes for an investment compounded annually to double in value is the ***Rule of 72*** . Dividing 72 by the annual interest rate expressed as a percentage, will give you the number of years it would take to double the value. For example, an investment with an interest rate of 3% per annum will double in $72/3 = 24\ years.$

In this section students will identify when to use each formula. They will practice calculating both simple and compound interest and know the effect of increasing the number of compounding periods.

SCO	**N4 Demonstrate an understanding of simple and compound interest.** [CN, ME, PS, T]

ACHIEVEMENT INDICATORS

- Solve a problem that involves simple interest, given any three of the four values in the formula *I=Prt.*
- Compare simple and compound interest, and explain their relationship.
- Solve, using the formula $A = P\left(1+\frac{r}{n}\right)^{nt}$, a contextual problem that involves compound interest.
- Explain, using examples, the effect of different compounding periods on calculations of compound interest.
- Estimate, using the *Rule of 72*, the time required for a given investment to double in value.

Suggested Instructional Strategies

- Students should be given plenty of practice determining which formula to use in which situation, and then in calculating the answers.

Suggested Questions (Q) and Activities (Act) for Instruction and Assessment

Q Calculate the yearly interest rate on a $500 loan that was borrowed for 6 months if the total amount of interest paid is $35. Use the formula:

$$Interest\ (I) = Principal\ Amount\ (P) \cdot Interest\ Rate\ (r) \cdot time(t)$$

$$I = Prt$$

Q *Good Deal Financial* is offering 4.4% interest, compounded monthly, on amounts in savings accounts over $10 000. Set up a chart or spreadsheet and determine how much interest you would have earned after 1 year for a $12 000 investment. Use the formula to calculate the interest for 10 years.

Q Ron and Ann have just won $100 000 in a lottery. They have a young daughter who will be two in August. Ron suggests that they buy a GIC at 2.3% with the $100 000 over a 16-year period. If they cash in the GIC after 16 years will they have enough interest to pay for their daughter's first year at university? First-year tuition is expected to grow 175% from the present $3500 in 16 years.

Q Sue and Rob each invest $5 000 for 6 years at a rate of 3.2% per annum. Sue chooses simple interest as she would like access to the interest each year, and Rob chooses compound interest. How much interest did each person earn over the 6 years?

SCO	**N5: Demonstrate an understanding of credit options, including: credit cards, and loans.** [CN, ME, PS, R]

[C] Communication **[T]** Technology	**[PS]** Problem Solving **[V]** Visualization	**[CN]** Connections **[R]** Reasoning	**[ME]** Mental Math and Estimation

Scope and Sequence of Outcomes

Grade Nine	Grade Ten	Grade Eleven
	N5: Demonstrate an understanding of credit options, including: credit cards, and loans.	**N2**: Analyze costs and benefits of renting, leasing and buying. *(FWM11)* **N3**: Analyze an investment portfolio in terms of interest rate, rate of return, total return. *(FWM11)* **N4**: Solve problems that involve personal budgets *(FWM11)* **N1**: Analyze costs and benefits of renting, leasing and buying. *(FM11)* **N2**: Analyze an investment portfolio in terms of interest rate, rate of return, total return. *(FM11)*

ELABORATION

Credit plays a big part in modern society and all students need to have a good understanding of the advantages and disadvantages of different credit options, with an emphasis on how to use credit effectively and how to avoid it becoming a burden.

Students should understand that different lending institutions offer availability to money at different costs to the consumer. Students should become aware that stores offer credit opportunities, some with their own credit card systems, others by having customers pay off the purchase price plus interest over a period of time. Usually store credit has high interest rates.

Students should explore the effect of **down payments**, length of borrowing period, varying interest rates on monthly payments, and total cost or **finance charge** over time.

Credit card companies provide another credit option. Students should study how to calculate the interest and monthly payments. They should also understand the difference in charges between a **cash advance** and a charge for a purchase.

Bank loans involve **principal amounts**, **amortization periods**, and interest rates (simple and compound). A bank **line of credit** is an approved loan amount that gives quick access to money. **Overdraft protection** also offers a form of short term loan, as it allows you to withdraw more money from your account than you have in it up to a certain amount.

Students should compare credit card purchases with short term loans. They should calculate the monthly payments for a loan, using formulas ($I = Prt$ and $A = P + I$), tables, and with appropriate technology.

It is important to provide authentic examples which are relevant to students. Application forms for credit cards can be found on-line, as well as store promotions.

SCO	**N5: Demonstrate an understanding of credit options, including: credit cards, and loans.** [CN, ME, PS, R]

ACHIEVEMENT INDICATORS

- Compare advantages and disadvantages of different types of credit options, including bank and store credit cards, personal loans, lines of credit, and overdraft.
- Make informed decisions and plans related to the use of credit, such as service charges, interest, payday loans and sales promotions, and explain the reasoning.
- Describe strategies to use credit effectively, such as negotiating interest rates, planning payment timelines, reducing accumulated debt and timing purchases.
- Compare credit card options from various companies and financial institutions.
- Solve a contextual problem that involves credit cards or loans.
- Solve a contextual problem that involves credit linked to sales promotions.

Suggested Instructional Strategies

- Have an expert from the community come in as a guest speaker. Students can prepare questions to ask the speaker after his or her presentation.
- Introduce the various credit options to students through scenario that are relevant to the student, such as a class trip, school make-over, buying a car, etc.
- Ensure topics are relevant and of current interest to student throughout this topic.
- Module 6 and 7 of "The City" provides a good resource for this outcome. (www.fcac-acfc.gc.cc)

Suggested Questions (Q) and Activities (Act) for Instruction and Assessment

Act Provide 3 authentic credit card applications and have students complete them. (from http://learningtogive.org/lessons/unit351/lesson4_attachments/2.html)

Q Sarah's credit card has a balance at the beginning of March of $37.05. She charged $51.60 and $427.75 on the card that month; and at the end of the month, interest was calculated on the unpaid balance from the beginning of March (12%), and she made the minimum payment (3% of balance).

In April Sarah used her credit card for purchases of $31.50, $10.60, and $17.25. For the end of April, calculate the interest charged, the minimum payment, and the balance left on the card.

Q Jarred can buy a used motorcycle for $2300 cash or pay a down payment of 10% and make 30 monthly payments of $78.25. Calculate the finance charge and percent rate.

Q For each of the following, calculate the total installment price, and the finance charge for paying for the article over time. Sales tax is 14%.

a) Electric Guitar: $279.98 or 15 monthly payments of $22.75

b) iPAD: $388.88 or $19 down payment plus 24 payments of $16.99.

SCO	**N5: Demonstrate an understanding of credit options, including: credit cards, and loans.** [CN, ME, PS, R]

Q Barbara buys a home theatre system for $3495. This price includes the N.B. taxes. She does not have the cash to buy it outright so she takes advantage of the credit offered by the store. She must pay an administration fee of $49.25 up front but no further payments for a year. After the year is up she will start to make 48 monthly payments at $85.25.

a) Find the total amount she will have paid for this living room set through store credit.

b) What is the store's finance charge?

c) What percent is the finance charge of the total payment?

d) Barbara could get a personal loan from the bank at an interest rate of 8% per year, over 4 years. Should she choose this option instead?

Q Use the following chart to summarize information on three different credit cards. Based on the information from the chart, discuss which credit card would you choose and why.

Card Costs and Features	Card 1:	Card 2:	Card 3:
Interest Rate			
Balance Calculation Method			
Duration of Grace Period			
Annual Fee			
Late Fee			
Cash Advance Fee			
Over the Limit Fee			
Transaction Fee			
Minimum Finance Charge			
Any Special Offers?			

Act Provide students with a sample credit card statement and ask them to answer the following questions:

a) What is the finance charge for the current billing period?

b) What is the annual percentage rate?

c) What is the minimum payment due?

d) What is the total amount charged for purchases made during the previous month?

e) How much credit does the card owner still have available?

f) What is the new balance?

g) How is the new balance determined?

SCO	**N5: Demonstrate an understanding of credit options, including: credit cards, and loans.** [CN, ME, PS, R]

Q Fill in the missing information for the credit card statement shown below. Interest is calculated at 2.5% per month.

Date	*Item*	*Amount*	*Amount Owing*
10/11/12	***Previous Balance***		
10/11/14	Toiletries	14.95	
10/11/27	Ladies' Wear	23.72	
10/11/30	Interest		286.70
10/12/02	Sporting Goods	48.73	
10/12/07	Payment	(------)	
10/12/08	Restaurant	27.58	

Balance Last Statement	**Total Credits**	**Total Debits**	**Total Interest**	**New Balance**
$243.17	$90.00	__________	____________	____________
Minimum Payment $13.65	*Due 10/06/12*			

Q Mary and David bought their first dining room suite from McDonald's Used Furniture. Consider the following options and decide which method of payment would be the most economical.

a) $100 down and 12 monthly payments of $125.00

b) a bank loan for $1200 at 14% annual interest compounded monthly

c) a credit card with a 19.9% annual interest rate and a minimum monthly payment of 4% of the unpaid balance.

Show all of your calculations to justify your decision.

SCO	**G1: Analyze puzzles and games that involve spatial reasoning, using a variety of problem-solving strategies.** [C, CN, PS, R]

Geometry

[C] Communication **[T]** Technology	**[PS]** Problem Solving **[V]** Visualization	**[CN]** Connections **[R]** Reasoning	**[ME]** Mental Math and Estimation

Scope and Sequence of Outcomes

Grade Nine	Grade Ten	Grade Eleven
	G1: Analyze puzzles and games that involve spatial reasoning, using a variety of problem-solving strategies.	**N1**: Analyze puzzles and games that involve numerical reasoning, using a variety of problem-solving strategies. *(FWM11)* **LR1**: Analyze and prove conjectures using logical reasoning to solve problems (*FM11)* **LR2**: Analyze puzzles and games that involve numerical reasoning, using problem-solving strategies *(FM11)*

ELABORATION

The focus in Grade 10 will be on spatial reasoning to solve puzzles and play games, throughout the course. However, it is not enough for students to only do the puzzle or play the game. They should be given a variety of opportunities to analyze the puzzles they solve and the games they play. The goal is to develop their problem-solving abilities using a variety of strategies and to be able to apply these skills to other contexts in mathematics. In Grade 11 the focus will shift to numerical reasoning.

ACHIEVEMENT INDICATORS

- Determine, explain and verify a strategy to solve a puzzle or to win a game:
  - guess and check
  - look for a pattern
  - make a systematic list
  - draw or model
  - eliminate possibilities
  - simplify the original problem
  - work backward
  - develop alternative approaches.
- Identify and correct errors in a solution to a puzzle or in a strategy for winning a game.
- Create a variation on a puzzle or a game, and describe a strategy for solving the puzzle or winning the game.

SCO **G1: Analyze puzzles and games that involve spatial reasoning, using a variety of problem-solving strategies.**
[C, CN, PS, R]

Suggested Instructional Strategies

- Choose a variety of games or puzzles for students to play online, with pencil and paper, or using models that require a variety of strategies to solve.
- Have students create a game or puzzle to challenge other students.
- Puzzles and discussions of strategies should be spread throughout the semester.

Suggested Activities for Instruction and Assessment

Act There are numerous games and puzzles available on the internet. What follows are just a few suggestions of spatial games and puzzles, available for free online. Many can also be done on paper, with models or acted out.

Note: It is expected that the teacher confirm the validity of the site prior to directing students to it.

Math is Fun site http://www.mathsisfun.com

Towers of Hanoi http://www.mathsisfun.com/games/towerofhanoi.html
Tic-Tac-Toe http://www.mathsisfun.com/games/tic-tac-toe.html
Dots and Boxes Game http://www.mathsisfun.com/games/dots-and-boxes.html
Four in a Line http://www.mathsisfun.com/games/connect4.html

NCTM at http://calculationnation.nctm.org/Games/

Nextu – In this game players alternate claiming shapes on a tessellation with shapes of greater value than the adjoining shape
Flip-n-Slide – This is a game in which triangles are translated and reflected to capture ladybugs. It is spatially challenging to envision the final position of the triangle.

Nrich at http://nrich.maths.org

Sprouts – is a game for two players, and can be played with a paper and a pencil. The rules are simple but the strategy can be complex. The site discusses the game, its history and strategies for solving the puzzle.
Frogs and Toads – is a well known puzzle in which frogs and toads must change places in as few turns as possible.

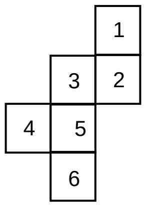
Jill Britton (personal site) http://britton.disted.camosun.bc.ca/nim.htm

Nim – is an ancient game that can be played online, on paper or using sticks – the winner is the player to not pick up the last stick (Common Nim) or to pick up the last stick (Straight Nim).

Cool Math http://www.coolmath-games.com/0-b-cubed/index.html
Cubed - is a game where students must pass over each block prior to reaching the final red block. Other spatial games on this site include pig stacks, aristetris, and bloxorz

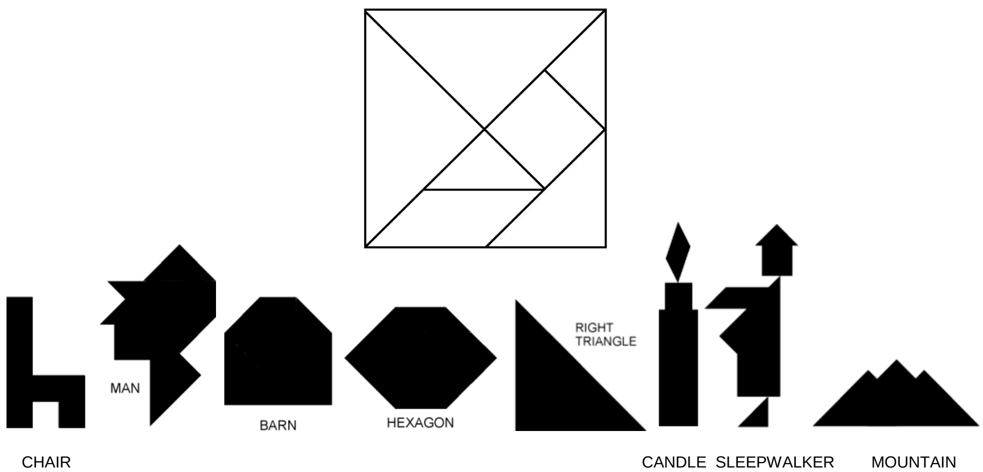
SCO **G1: Analyze puzzles and games that involve spatial reasoning, using a variety of problem-solving strategies.**
[C, CN, PS, R]

Q If the numbered net shown below is folded to form a cube, what is the product of the numbers on the four faces sharing an edge with the face number 1?



Q The tangram puzzle was invented in China thousands of years ago. The object is to arrange all seven pieces of the tangram (cut from a square as shown below), into various shapes just by looking at the outline of the solution. Try making the shapes shown below, using all seven pieces.

Design other puzzles and challenge your classmates to solve them.



SCO	**G2: Demonstrate an understanding of the Pythagorean theorem by: identifying situations that involve right triangles, verifying the formula, applying the formula, and solving problems.** [C, CN, PS, V]

[C] Communication **[T]** Technology	**[PS]** Problem Solving **[V]** Visualization	**[CN]** Connections **[R]** Reasoning	**[ME]** Mental Math and Estimation

Scope and Sequence of Outcomes

Grade Nine	Grade Ten	Grade Eleven
SS1: Solve problems and justify the solution strategy using circle properties including: the perpendicular from the centre of a circle to a chord bisects the chord; the measure of the central angle is equal to twice the measure of the inscribed angle subtended by the same arc; the inscribed angles subtended by the same arc are congruent; a tangent to a circle is perpendicular to the radius at the point of tangency. (triangular prisms) **SS2:** Determine the surface area of composite 3-D objects to solve problems. **SS4:** Draw and interpret scale diagrams of 2-D shapes.	**G2:** Demonstrate an understanding of the Pythagorean theorem by: identifying situations that involve right triangles, verifying the formula, applying the formula, and solving problems.	**G1**: Solve problems that involve two and three right triangles. *(FWM11)* **G3 :** Solve problems that involve the cosine law and the sine law, including the ambiguous case. *(FM11)* **T2 :** Solve problems, using the tree primary trigonometric ratios for angles from $0°$ to $360°$ in standard positions. *(PC11)*

ELABORATION

Students were introduced to the Pythagorean Theorem in Grade 8. They should know that the theorem only applies to right triangles and should be able to solve problems that involve Pythagorean triples.

Students will further explore the development, conditions and practical applications of the Pythagorean Theorem.

ACHIEVEMENT INDICATORS

- Explain, using illustrations, why the Pythagorean Theorem only applies to right triangles.
- Verify the Pythagorean Theorem, using examples and counterexamples, including drawings, concrete materials and technology.
- Describe historical and contemporary applications of the Pythagorean Theorem.
- Determine if a given triangle is a right triangle, using the Pythagorean Theorem.
- Explain why a triangle with the side length ratio of 3:4:5 is a right triangle.
- Show how a Pythagorean triple can be used to determine if a corner of a given 3-D object is square (90º) or if a given parallelogram is a rectangle.
- Solve a problem, using the Pythagorean Theorem.

SCO	**G2: Demonstrate an understanding of the Pythagorean theorem by: identifying situations that involve right triangles, verifying the formula, applying the formula, and solving problems.** [C, CN, PS, V]

Suggested Instructional Strategies

- Explore the multiple ways of proving Pythagorean Theorem concretely, pictorially and symbolically. Consider using online models as well as historical examples (e.g. Egyptian)

  Current sites include a Youtube video which uses origami to investigate the Pythagorean theorem, and a teacher's site which has examples from a variety of ancient texts including a medieval European military handbook, an Egyptian mathematical papyrus (300 BCE), a Mesopotamian clay table (1900-1600 CE), an Indian mathematician and astronomer (1150 BCE), and a Chinese text (200 BCE). http://bulldog2.redlands.edu/fac/beery/math115/day4.htm

  (Note: Teachers must always check the current validity of an online site prior to directing students to it)

- Use counter-examples so that students can discover the Pythagorean Theorem works only with 90º angles.

Suggested Questions (Q) and Activities (Act) for Instruction and Assessment

Act In small groups, have students develop a question, and then switch between groups and solve.

Act Have students research historical applications of the Pythagorean Theorem**.**

Q Is a triangle with dimensions $10cm \times 12cm \times 15cm$ a right triangle? Justify your answer.

Q Prove that 15 - 20 - 25 are the sides of a right angle triangle. How do you know which side is the hypotenuse?

Q Sketch and then solve the following problem:

A $13\ m$ long wheel chair ramp, leading to an entrance, is being constructed. If the top of the ramp is $4.5\ m$ off the ground, determine the length of the ramp sitting on the driveway.

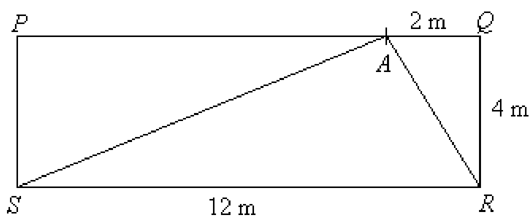
Q Joanne was given a triangle and asked if it was a right triangle. She felt it was, so she set out to prove it by measuring the sides and using Pythagorean Theorem. Her work is shown below. Did she prove this was a right triangle? Why or why not?

$c^2 = a^2 + b^2$
$10^2 = 6^2 + 9^2$
$100 = 36 + 81$
$100 = 117$

Q A ship leaves port and sails 20km north and then 13 km east. How far is the ship from port?

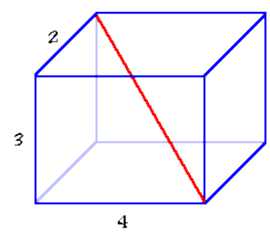
SCO	**G2: Demonstrate an understanding of the Pythagorean theorem by: identifying situations that involve right triangles, verifying the formula, applying the formula, and solving problems.** [C, CN, PS, V]

Q The rectangle *PQRS* represents the floor of a room.



Sarah stands at point *A*. Calculate her distance from:
a) the corner *R* of the room
b) the corner *S* of the room

Q A wooden box measures $4\text{m} \times 3\text{m} \times 2\text{m}$. What is the longest straight pole? (shown in the picture) that can fit from one corner to the opposite corner inside the box?



Q In a flyer, a TV is listed as being $55\ inches$. This distance is the diagonal distance across the screen. If the screen measures $28\ inches$ in height, will the TV fit on my TV stand, which is $48\ inches$ wide?

SCO	**G3: Demonstrate an understanding of primary trigonometric ratios (sine, cosine, tangent) by: applying similarity to right triangles, generalizing patterns from similar right triangles, applying the primary trigonometric ratios, and solving problems.** [CN, PS, R, T, V]

[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Math **[T]**
Technology	**[V]** Visualization	**[R]** Reasoning	and Estimation

Scope and Sequence of Outcomes

Grade Nine	Grade Ten	Grade Eleven
SS4 Draw and interpret scale diagrams of 2-D shapes. (similar triangles)	**G3:** Demonstrate an understanding of primary trigonometric ratios (sine, cosine, tangent) by: applying similarity to right triangles, generalizing patterns from similar right triangles, applying the primary trigonometric ratios, and solving problems.	**G1** Solve problems that involve two and three right triangles. *(FWM11)* **G3**: Solve problems that involve the cosine law and the sine law, including the ambiguous case *(FM11)* **T2 :** Solve problems, using the tree primary trigonometric ratios for angles from $0°$ to $360°$ in standard positions. *(PC11)*

ELABORATION

This is the first time that students have seen trigonometric ratios in the mathematics curriculum. Their exploration will be limited to the primary trigonometric ratios with respect to right angle triangles.

Students will identify the opposite, adjacent and hypotenuse sides with reference to one of the acute angles in a given right triangle. They will recognize that the opposite and adjacent sides change dependent on the angle chosen. They will define each of the primary trigonometric ratios and explore the trend of the ratio as the angles change.

To solve problems involving trigonometric ratios students will learn to recognize which ratio to apply in a given situation, to solve for the unknown variable, and will confirm that the answer is reasonable.

Students will also need to be proficient at rearranging a formula for a given variable. This is one of the applications from *SCO A1*.

SCO **G3: Demonstrate an understanding of primary trigonometric ratios (sine, cosine, tangent) by: applying similarity to right triangles, generalizing patterns from similar right triangles, applying the primary trigonometric ratios, and solving problems.**
[CN, PS, R, T, V]

ACHIEVEMENT INDICATORS

- Show, for a specified acute angle in a set of similar right triangles, that the ratios of the length of the side opposite to the length of the side adjacent are equal, and generalize a formula for the tangent ratio.
- Show, for a specified acute angle in a set of similar right triangles, that the ratios of the length of the side opposite to the length of the hypotenuse are equal, and generalize a formula for the sine ratio.
- Show, for a specified acute angle in a set of similar right triangles, that the ratios of the length of the side adjacent to the length of the hypotenuse are equal, and generalize a formula for the cosine ratio.
- Identify situations where the trigonometric ratios are used to determine measurement of angles and lengths indirectly e.g. determining the height of a flagpole, or a tree from the ground.
- Solve a contextual problem using the primary trigonometric ratios that involves right triangles.
- Determine if a solution to a problem that involves primary trigonometric ratios is reasonable.

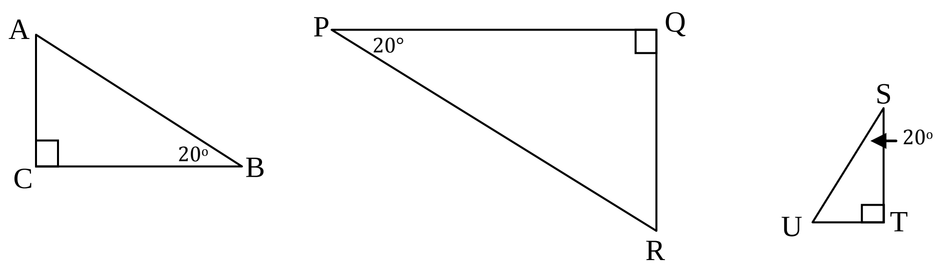
Suggested Instructional Strategies

- Allow students to discover the trigonometry ratios through angle and side length measurements, and calculation of ratio values for triangles with common angle measurements.
- Use angle of elevation and angle of depression to help create contextual problems for this outcome.
- Use clinometers and have students go outside to find the height of tall buildings or trees using trigonometric ratios.
- Work with students to create diagrams that are labeled correctly from word problems (a skill that many students struggle with).

SCO	**G3: Demonstrate an understanding of primary trigonometric ratios (sine, cosine, tangent) by: applying similarity to right triangles, generalizing patterns from similar right triangles, applying the primary trigonometric ratios, and solving problems.** [CN, PS, R, T, V]

Suggested Questions (Q) and Activities (Act) for Instruction and Assessment

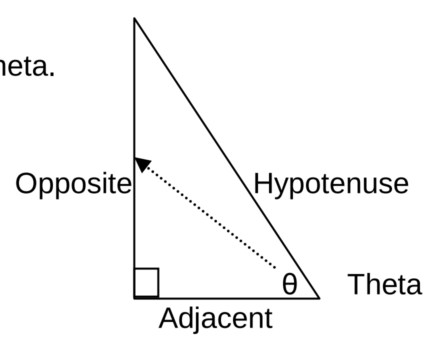
Act Allow students to discover the trigonometry ratios through completing the following or a similar chart for three similar triangles with a common angle measurement.



Triangle	For 20° angle	Length of opposite	Length of adjacent	Length of hypotenuse	$\frac{O}{H}$	$\frac{A}{H}$	$\frac{O}{A}$
ABC	*B*						
PQR	*P*						
STU	*S*						
				Mean Values			

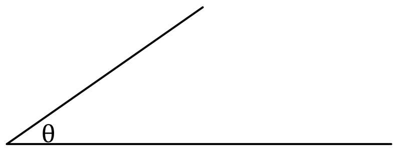
Act Create 5 large right-angled triangles on the floor with masking tape. Then divide your class into 5 groups. As the students stand around the triangles each member of the group receives one of the following words or symbols: "opposite", "adjacent"," hypotenuse", "theta", "θ" (symbol for theta), and a 5 cm x 5 cm square.

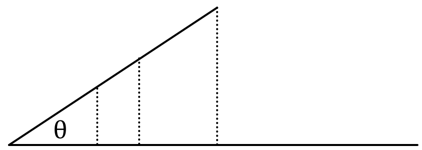
1. Have students identify the type of triangle on the floor by indicating the right angle with the square and then label the hypotenuse.
2. Have the person holding the symbol for theta place it in one of the other angles of the triangle. The word "theta" is then placed above it as reinforcement for the new terminology.
3. The "opposite" person then goes to theta and walks across the triangle to get to the "opposite" side and places it accordingly.
4. Adjacent is then placed on the side next to theta.
5. Finally, students are instructed to move Theta to the other angle in the triangle. At that point opposite and adjacent must be relocated but hypotenuse remains in place.



SCO	**G3: Demonstrate an understanding of primary trigonometric ratios (sine, cosine, tangent) by: applying similarity to right triangles, generalizing patterns from similar right triangles, applying the primary trigonometric ratios, and solving problems.** [CN, PS, R, T, V]

Act Give students an angle of 30°. Label the angle as *theta* (θ) and have them construct a series of right-angled triangles at various distances from the angle.





Students should then measure the sides of each right-angled triangle and fill out the chart below for each triangle.

Opposite	Adjacent	Hypotenuse	Sine	Cosine	Tangent

If the data from all students can be entered into an *Excel* file, the sine, cosine and tangent values can be automatically calculated and students will see that their sine, cosine and tangent values are all very similar (discuss why they may differ), if not identical to their classmates. This will lead to the discussion of the *Trigonometric Table* and how it can be used to determine that the value of *theta* is $30°$.

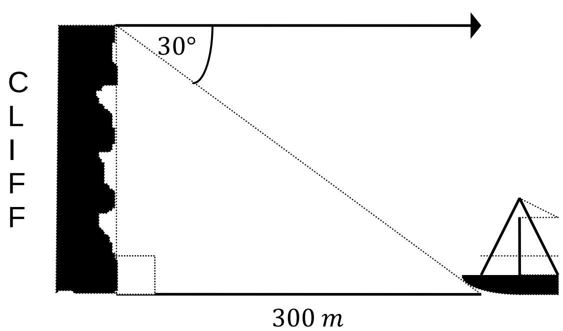
Q Verify if the following two formulas are equivalent:

$\mathbf{sin}\,\boldsymbol{\theta} = \frac{\text{opposite}}{\text{hypotenuse}}$ and $hypotenuse = \frac{opposite}{\sin\theta}$

Q Create a contextual problem which could be solved using the following formula:

$$\mathbf{cos}\,\boldsymbol{\theta} = \frac{\boldsymbol{adjacent}}{\boldsymbol{hypotenuse}}$$

Q The angle of depression from the top of a cliff to a sailboat below is $30°$. If the sailboat is $300\ m$ from the cliff what is the height of the cliff?



Q Michael has a summer job with a company that builds antenna towers. He needs to determine the length of cable needed to stabilize a 30 m tower. The cable must make a 65° angle with the ground. Draw a labelled diagram and calculate the length of the cable required.

SCO	**G4: Solve problems that involve angle relationships between parallel, perpendicular and transversal lines.** [C, CN, PS, V]

[C] Communication **[T]** Technology	**[PS]** Problem Solving **[V]** Visualization	**[CN]** Connections **[R]** Reasoning	**[ME]** Mental Math and Estimation

Scope and Sequence of Outcomes

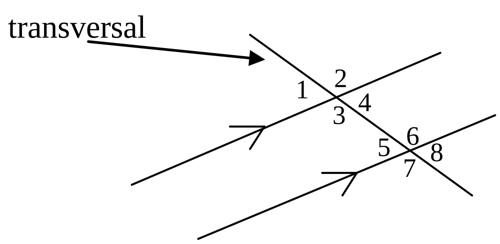
Grade Nine	Grade Ten	Grade Eleven
SS1: Solve problems and justify the solution strategy using circle properties, including: the perpendicular from the centre of a circle to a chord bisects the chord; the measure of the central angle is equal to twice the measure of the inscribed angle subtended by the same arc; the inscribed angles subtended by the same arc are congruent; a tangent to a circle is perpendicular to the radius at the point of tangency.	**G4:** Solve problems that involve angle relationships between parallel, perpendicular and transversal lines.	**G1** Solve problems that involve two and three right triangles. *(FWM11)* **G1:** Derive proofs that involve the properties of angles and triangles. *(FM11)* **G2**: Solve problems that involve the properties of angles and triangles. (*FM11)*

ELABORATION

Students have explored the concepts of parallel and perpendicular lines in previous grades. Students will now extend this learning to explore and categorize the relationship between two lines as parallel, perpendicular or neither, and to determine the relationships between the angles formed in various contexts.

Students will be introduced to the concepts of **complementary** (adding to 90°), and **supplementary** (adding to 180°) angles.

Transversals of parallel lines and the resulting angle relationships will be investigated. Pairs of angles in this context can be **congruent** or supplementary.



With reference to the figure above, the following angles are congruent to each other:
∠1, ∠4, ∠5 and ∠8
∠2, ∠3, ∠6 and ∠7
This includes congruence between pairs of:
vertically opposite angles ∠1 and ∠4, ∠2 and ∠3, ∠5 and ∠8, ∠6 and ∠7
corresponding angles ∠1 and ∠5, ∠3 and ∠7, ∠2 and ∠6, ∠4 and ∠8
alternate interior angles ∠3 and ∠6, ∠4 and ∠5
alternate exterior angles ∠1 and ∠8, ∠2 and ∠7

With reference to the figure above, the following pairs of angles are supplementary:
interior angles on the same side of the transversal ∠3 and ∠5, ∠4 and ∠6
exterior angles on the same side of the transversal ∠2 and ∠8, ∠1 and ∠7

SCO	**G4: Solve problems that involve angle relationships between parallel, perpendicular and transversal lines.** [C, CN, PS, V]

These angle relationships hold when lines are parallel (Figure 1), However, when lines are not parallel (Figure 2), the angle relationships do not apply, except for the congruency of opposite angles.

This can be explained by recognizing that if the lines are not parallel they will eventually meet, forming a triangle with the transversal. In the example shown below, angles 3 and 5 are no longer supplementary as they are now two angles in a triangle and $\angle 3 + \angle 5 + \angle 9 = 180°$. Although opposite angles remain congruent as for any two lines crossing, the other angle relationships no longer apply.

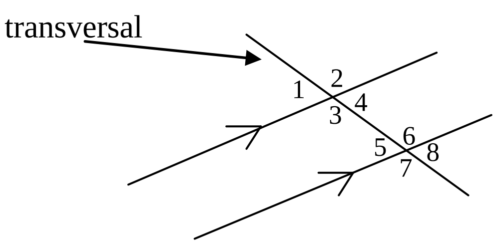


Figure 1

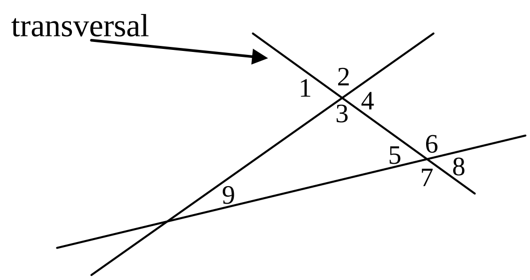


Figure 2

SCO **G4: Solve problems that involve angle relationships between parallel, perpendicular and transversal lines.**
[C, CN, PS, V]

ACHIEVEMENT INDICATORS

- Sort a set of lines as perpendicular, parallel or neither, and justify this sorting.
- Illustrate and describe complementary and supplementary angles.
- Identify, in a set of angles, adjacent angles that are not complementary or supplementary.
- Identify and name pairs of angles formed by parallel lines and a transversal, including corresponding angles, vertically opposite angles, alternate interior angles, alternate exterior angles, interior angles on same side of transversal and exterior angles on same side of transversal.
- Explain, using examples, why the angle relationships do not apply when the lines are not parallel.
- Determine the measures of angles involving parallel lines and a transversal, using angle relationships.
- Solve a contextual problem that involves angles formed by parallel lines and a transversal (including perpendicular transversals).

Suggested Instructional Strategies

- Have students create a legend of rules for angles formed when a transversal intersects parallel lines.
- Have students create their own question with parallel lines and a transversal and then have them solve it.

Suggested Questions (Q) and Activities (Act) for Instruction and Assessment

Act Give students a map (e.g. a city in NB) and ask them to find examples of the following: parallel lines, perpendicular lines, transversal lines, complementary angles, supplementary angles etc.

SCO **G5: Demonstrate an understanding of angles, including acute, right, obtuse, straight and reflex, by: drawing, replicating and constructing, bisecting, and solving problems.**
[C, ME, PS, T, V]

[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Math **[T]**
Technology	**[V]** Visualization	**[R]** Reasoning	and Estimation

Scope and Sequence of Outcomes

Grade Nine	Grade Ten	Grade Eleven
	G5: Demonstrate an understanding of angles, including acute, right, obtuse, straight and reflex, by: drawing, replicating and constructing, bisecting, and solving problems.	**G1** Solve problems that involve two and three right triangles. *(FWM11)* **G2**: Solve problems that involve the properties of angles and triangles. (*FM11)* **T1**: Demonstrate an understanding of angles in standard position $[0^\circ - 360^\circ]$. (*PC11)*

ELABORATION

Students have studied the concept of angles since Grade 6. In Grade 7 students performed geometric constructions including: perpendicular line segments; parallel line segments; perpendicular bisectors; and angle bisectors.

Students will explore angles from a variety of perspectives. They will sketch, describe and estimate angles with an understanding of the referent angles; 30, 45, 60, 90 and 180 degrees. Students will also use a variety of tools to measure, replicate and bisect angles.

These skills will be applied to contextual problems e.g. in navigation, orientation and construction.

Students will be required to use a variety of tools. Teachers should be aware that there is an add-on for Smart notebook 10 called Math Tools which will enable teachers to perform constructions on the Smartboard.

SCO **G5: Demonstrate an understanding of angles, including acute, right, obtuse, straight and reflex, by: drawing, replicating and constructing, bisecting, and solving problems.**
[C, ME, PS, T, V]

ACHIEVEMENT INDICATORS

- Draw, describe and identify angles with various measures, including acute, right, straight, obtuse and reflex angles.
- Estimate the measure of a given angle, using 30°, 45°, 60°, 90° and 180° as referent angles.
- Sketch an angle, based on referent angles
- Measure, using a protractor, angles in various orientations.
- Explain and illustrate how angles can be replicated in a variety of ways; Mira, protractor, compass and straightedge, carpenter's square, dynamic geometry software.
- Replicate angles in a variety of ways, with and without technology.
- Bisect an angle, using a variety of methods.
- Solve a contextual problem that involves angles.

Suggested Instructional Strategies

- This outcome allows for many opportunities for students to construct and replicate their own angles using a variety of tools. Ensure that students have access to the tools listed in this document.
- Have students list examples of real life situations where they see angles that are acute, right, obtuse, straight and reflex on a clock face, in furniture, roof slopes, in art etc. With this list have them submit 2 or 3 digital pictures for each of these examples (use their cameras on their phone). They could submit this as project.

SCO	**G5: Demonstrate an understanding of angles, including acute, right, obtuse, straight and reflex, by: drawing, replicating and constructing, bisecting, and solving problems.** [C, ME, PS, T, V]

<u>Suggested Questions (Q) and Activities (Act) for Instruction and Assessment</u>

Q Use an analogue clock face, and with a partner identify 5 times that you would find each of the following types of angles: acute, right, obtuse, straight and reflex angles.

a) What is the approximate degree measure for each of these angles?
b) What time would it be if you doubled each of these angles?
c) What time would it be if you bisected each of these angles?

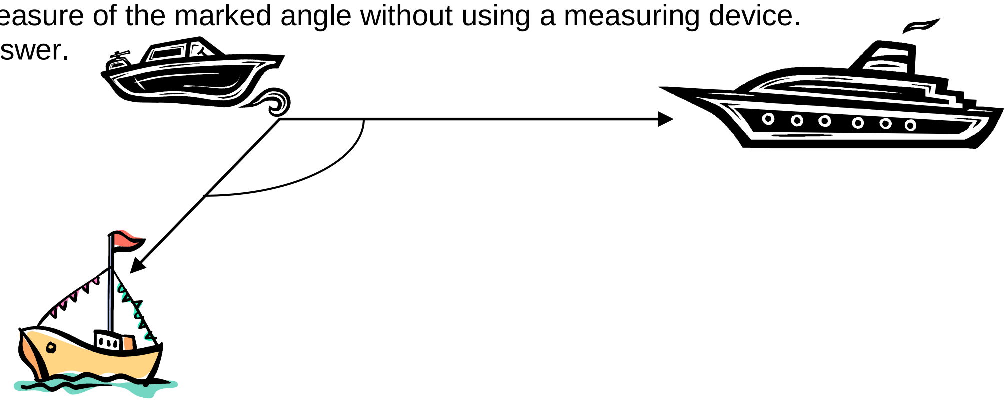
(note to teacher: Students should recognize the movement of the hour hand as well as the minute hand and can be encouraged to figure out the exact angle measure e.g. at 9:00, the hands are exactly $90°$ apart but at 9:15 the hour hand is ¼ of the way to the 10: $360° \div 12 \div 4 = 7.5°$ $\therefore$ *the angle between the hour and minute hand at* $9{:}15$ *is* $180° - 7.5° = 172.5°$).

Q Find examples in the classroom, school, community and at home of real life situations where you see angles that are acute, right, obtuse, straight and reflex. As an assignment, submit 2-3 digital pictures for each of these examples, accompanied by a description of where you found it, what it is, the angle type and the approximate angle measure.

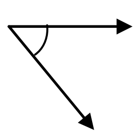
Q During a home renovation project, Adam decides to put new moulding around the doors. He cuts the top piece of the door moulding at 45° angles. If Adam wants 90°corners on the mouldings, at what angles must he cut the top ends of the side pieces?

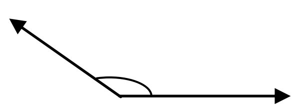
Q If the hour hand moves 210° clockwise, from the 12, at what hour is the hand pointing?

Q Estimate the measure of the marked angle without using a measuring device. Explain your answer.



Q Estimate the measure of each angle shown below and the measure of each of the angles bisected.

a) 

b) 

SCO	**M1: Demonstrate an understanding of the Système International (SI) by: describing the relationships of the units for length, area, volume, capacity, mass and temperature.** [C, CN, ME, V]

Measurement

[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Math and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Scope and Sequence of Outcomes

Grade Nine	Grade Ten	Grade Eleven
SS2: Determine the surface area of composite 3-D objects to solve problems.	**M1**: Demonstrate an understanding of the Système International (SI) by: describing the relationships of the units for length, area, volume, capacity, mass and temperature	**A3**: Solve problems by applying proportional reasoning and unit analysis. *(FWM11)*

ELABORATION

Students are familiar with the metric system (the SI system) as a standard for measurement. Students were introduced to the basic SI units in Grade 3 and extended their knowledge since that time to other units and to conversions within the SI system. Students should be able to identify commonly used SI units such as centimetres, metres, millilitres, litres, grams and kilograms.

Students will explore the origins of SI system, and its history and use in Canada. The metric system was formally developed by France in the 1700's. The system is based on the linear measure of a metre which was defined in terms of the Earth's circumference, and on base 10. Mass was based on the gram, defined as the mass of one cubic centimetre of water.

In Canada in 1970, with rapidly advancing technology and expanding worldwide trade, the Canadian government adopted a policy for a single, coherent measurement system based on the ***Système International d'Unités* (SI)**, the latest evolution of the metric system.

To fully understand measurement, students will need to distinguish which measures are metric and which are Imperial. This outcome focuses on first developing proficiency with the metric system.

For this outcome students should recognize the terminology and abbreviations associated with SI units such as: metre (mm, cm, m, km), litre (mL, L), hectare (ha), degree Celsius$(℃)$, gram(mg, g, kg).

Students should also be able to identify when SI measures are most commonly used such as: fuel economy $(L/100km)$, meat or fish (kg), milk or juice (L), cloth (m), boiling point of water $(100℃)$, freezing point of water $(0℃)$, summer air temperature $(30℃)$, carpet lengths(m), volumes of beakers in science class $(100\,mL,\ 250\,mL)$.

Students will convert between various units in the SI system. Students have used and will continue to use conversions in science.

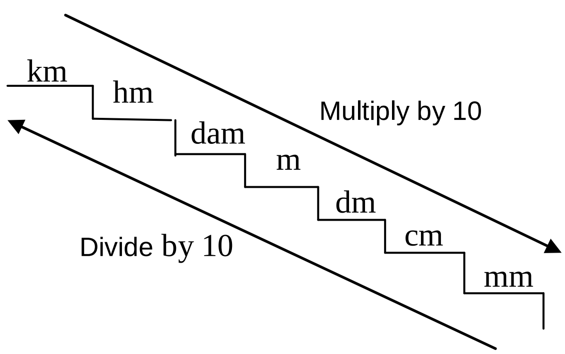
SCO	**M1: Demonstrate an understanding of the Système International (SI) by: describing the relationships of the units for length, area, volume, capacity, mass and temperature.** [C, CN, ME, V]

ACHIEVEMENT INDICATORS

- Explain how the SI system was developed, and explain its relationship to base ten.
- Identify the base units of measurement in the SI system, and determine the relationship among the related units of each type of measurement.
- Identify contexts that involve the SI system.
- Match the prefixes used for SI units of measurement with the powers of ten.
- Explain, using examples, how and why decimals are used in the SI system.
- Write a given measurement expressed in one SI unit, in another SI unit.

<u>Suggested Instructional Strategies</u>

- Activate prior knowledge, regarding referents of measurement which students have previously developed (1 metre = floor to door knob, 1 litre of milk, room temperature 21^{o}C, 2 lbs of sugar, 1 kg of salt).
- It could be useful for teachers to emphasize the ease of conversion between metric units given that it is developed on the base 10 model. This makes conversions easier with the SI system than the Imperial system.
- For length measurements (not area or volume) the following step model may help students visualize converting within the SI measurement system. Each step represents multiplication or division by a factor of 10.



- Proportional reasoning skills, covered in *SCO N1,* should be introduced here as one way to complete conversions within the SI system. For example: to convert 0.03 kilometres to metres, solve for ?:

$$\frac{1000\ m}{1\ km} = \frac{?\ m}{0.03\ km}$$

$$\frac{1000m\ \times 0.03\ km}{1\ km} = ?\ m$$

$$x = 30$$

$$\therefore\ 0.03\ km = 30\ m$$

SCO	**M1: Demonstrate an understanding of the Système International (SI) by: describing the relationships of the units for length, area, volume, capacity, mass and temperature.** [C, CN, ME, V]

Suggested Questions (Q) and Activities (Act) for Instruction and Assessment

Q Define the prefixes used in the SI system. Include mega-, giga-, tera- and others. In what other context do you see these prefixes being used?

Q Jason was asked how many iPod charging cords it would take to go around the perimeter of the classroom. He measured the length of the cord to be$75\ cm$. Knowing $100\ cm$ was in a meter, he did a conversion resulting in$75\ cm = 7500\ m$.

a) Is Jason's answer correct? Why or why not?
b) If the classroom measures *6 m x 5.5 m*, how many cords would it take to go around the classroom?

Q Fill in the blanks:

a) $130\ cm$ = ________ m
b) _______ g = $150\ mg$
c) $60\ L$ = _______mL
d) $3.25\ km$ = _______cm
e) _______ g = $0.68\ kg$
f) $4m^2$= _______ cm^2
g) $3cm^2$=_______ mm^3

Act Have students measure the mass, volume, capacity and temperature using different measurement tools (metre sticks, measuring tapes, callipers). Convert the readings to other units which might be appropriate. For example: measuring the thickness of a sheet of paper with callipers will illustrate the usefulness of mm rather than cm or m; measuring the volume of a milk or juice carton, will illustrate the usefulness of using L rather than mL.

Q Serena used proportional reasoning to do the conversion: $0.78\ kg =$ ____mg

She wrote: $$\frac{10\ 000\ mg}{1\ kg} = \frac{?\ mg}{0.78\ kg}$$

Where did she make an error? Complete the conversion correctly.

SCO	**M2: Demonstrate an understanding of the Imperial system by: describing the relationships of the units for length, area, volume, capacity, mass and temperature.** [C, CN, ME, V]

[C] Communication **[T]** Technology	**[PS]** Problem Solving **[V]** Visualization	**[CN]** Connections **[R]** Reasoning	**[ME]** Mental Math and Estimation

Scope and Sequence of Outcomes

Grade Nine	Grade Ten	Grade Eleven
SS2 Determine the surface area of composite 3-D objects to solve problems.	**M2:** Demonstrate an understanding of the Imperial system by: describing the relationships of the units for length, area, volume, capacity, mass and temperature.	**A3**: Solve problems by applying proportional reasoning and unit analysis. *(FWM11)*

ELABORATION

Students have been using the metric system in previous grades. This will be their first opportunity to study the **Imperial system**. Students may be familiar with the Imperial system for measurements such as distance in miles, height in feet and inches, weight in pounds and capacity in gallons.

Students will explore the origins of the Imperial system and its history and use in Canada. Although Canada officially adopted the metric system in 1970, Imperial measures continue to be used extensively. To fully understand measurement students will need to distinguish which measures are Imperial and which are metric. This outcome focuses on developing proficiency with the Imperial system and converting units within the Imperial system.

For this outcome students should recognize the terminology and abbreviations associated with Imperial measure such as: foot (ft or ‘), inch (in or “), yard (yd), miles (mi), pints (pt), quarts (qt), gallons (gal), teaspoon (tsp), tablespoon ($Tbsp$), pound (lb), ounces (oz), degrees Fahrenheit ($°\mathrm{F}$), acres(ac).

Students should also be able to identify when Imperial measures are most commonly used such as: cooking ($tsp, Tbsp, c, lb$), wood products ($2\ by\ 4$), fuel economy (mpg), pant leg length ($31"$), TV screen size ($28"$), paper size ($8.5'' \times 11''$), photograph size ($5'' \times 7''$), newborn’s weight ,($7lbs\ 6\ oz$), floor tile ($1\ sq\ ft$), room temperature ($68°\mathrm{F}$), freezing water ($32°\mathrm{F}$), height ($5'5''$), healthy body temperature ($98.6°\mathrm{F}$). lot size ($1\ acre\ house\ lot$).

Students will convert between the commonly used Imperial units for linear measure, area, capacity and temperature. They should know some basic conversion equivalencies such as: $12\ in = 1\ ft$, $3\ ft = 1\ yd$, $1\ c = 8\ fl\ oz$, $1\ lb = 16\ oz$, $4\ Tbsp = ¼\ c$. Other conversions need not be memorized, as they are easily accessible. Using the Imperial system will provide practice using fractions.

SCO	**M2: Demonstrate an understanding of the Imperial system by: describing the relationships of the units for length, area, volume, capacity, mass and temperature.** [C, CN, ME, V]

ACHIEVEMENT INDICATORS

- Explain how the Imperial system was developed.
- Identify commonly used units in the Imperial system, and determine the relationships among the related units.
- Identify contexts that involve the Imperial system.
- Explain, using examples, how and why fractions are used in the Imperial system.
- Write a given measurement expressed in one Imperial unit, in another Imperial unit.

Suggested Instructional Strategies

- When converting between Imperial units, have students practice proportional reasoning.
- Hands on activities will allow students to be more engaged for this outcome and develop skills for using measurement tools such as a measuring tape.
- Use of fractions can be demonstrated through cooking or construction. For example:
  - Halving or doubling a recipe
  - Measuring $16\frac{1}{8}$" in construction.

SCO	**M2: Demonstrate an understanding of the Imperial system by: describing the relationships of the units for length, area, volume, capacity, mass and temperature.** [C, CN, ME, V]

Suggested Questions (Q) and Activities (Act) for Instruction and Assessment

Act Lead a discussion about why the Imperial system is still used in Canada even though it is not the official system.

Act Use flyers or the internet to investigate products from building supply stores that show the use of Imperial units for measurements. Have students examine what material or objects are measured in the Imperial system and which ones are measured in SI system. Students will then record their finding and identify each measurement as to whether it is for length, area, volume, capacity, mass or temperature.

Act Using a measuring tape with inches and feet, have students measure 10 objects around the classroom to the closest one eighth of an inch.

Q Fill in the blanks:

a) $36\ in = \_\_\_ ft$ b) $6" = \_\_\_\ '$ c) $6\ ft = \_\_\_ yd$

d) $\_\_\_\_\_\ in = 2ft$ e) $1\ yd^2 = \_\_\_\_ ft^2$

Q A baby is born weighing 7 pounds and 8 ounces. The baby announcements in the newspaper said the baby weighed 7.5 lbs. Given this, how many ounces are in a pound?

SCO	**M3: Solve problems, using SI and Imperial units, that involve linear measurement using estimation and measurement strategies.** [ME, PS, V, C].

C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Math
[T] Technology	**[V]** Visualization	**[R]** Reasoning	and Estimation

Scope and Sequence of Outcomes

Grade Nine	Grade Ten	Grade Eleven
	M3: Solve problems, using SI and Imperial units, that involve linear measurement using estimation and measurement strategies.	**A3**: Solve problems by applying proportional reasoning and unit analysis. *(FWM11)*

ELABORATION

Students will compare, estimate and justify their choice of measurement system and units based on referents such as centimetres and inches.

Students will solve problems that involve linear measure, with a variety of tools such as rulers, callipers and tape measures.

Students will use both the Imperial and SI systems and convert between them as appropriate to the application. The main focus of this outcome is linear measurement, however, it is also important that students explore how to convert between pounds and kilograms.

Students will apply proportional reasoning to solve, verify and justify problems involving conversion within or between SI and Imperial units.

SCO **M3: Solve problems, using SI and Imperial units, that involve linear measurement using estimation and measurement strategies.**
[ME, PS, V, C].

ACHIEVEMENT INDICATORS

- Provide referents for linear measurements, including millimetre, centimetre, metre, kilometre, inch, foot, yard and mile, and explain the choices.
- Compare SI and Imperial units, using referents.
- Estimate a linear measure, using a referent, and explain the process used.
- Justify the choice of units used for determining a measurement in a problem-solving context.
- Solve problems that involve linear measure, using instruments such as rulers, callipers or tape measures.
- Describe and explain a personal strategy used to determine a linear measurement such as circumference of a bottle, length of a curve, or perimeter of the base of an irregular 3-D object.
- Solve a problem that involves the conversion of units within or between SI and Imperial systems.
- Verify, using unit analysis, a conversion within or between SI and Imperial systems, and explain the conversion.
- Justify, using mental mathematics, the reasonableness of a solution to a conversion problem.

SCO **M3: Solve problems, using SI and Imperial units, that involve linear measurement using estimation and measurement strategies.**
[ME, PS, V, C].

Suggested Instructional Strategies

- Set up an investigation where students must estimate and then measure several items. Have them convert the measurements to Imperial and then verify using an appropriate tool. The items provided for the students should be regular as well as irregular.
- Have students develop a set of referents, for both metric and Imperial commonly used measures such as metre, gram, inch and mile, and then use them to estimate the length of an unknown object. Some examples are shown below.

  Metre/Yard*:* height of a door knob from the floor

  Millimetre**:** the thickness of a dime

  Centimetre/ inch**:** width / length between 1st and 2nd knuckles of a baby finger

  Kilometre: distance you could walk comfortably in 12 minutes

  Gram: weight of a jelly bean

  Pound/ Kilogram: one/ two footballs

  Litre**:** small carton of milk

  Millilitre**:** the liquid you could fit in a base-10 unit cube

  Centigrade/ Fahrenheit: $20℃/\ 68℉$ room temperature

  Foot: foot long sub sandwich

- Have students measure, in Imperial and in metric, their hand span, foot length, finger length, and stride length and use these as referents to measure the length of the classroom, width of a desk etc.

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SCO	**M3: Solve problems, using SI and Imperial units, that involve linear measurement using estimation and measurement strategies.** [ME, PS, V, C].

Suggested Questions (Q) and Activities (Act) for Instruction and Assessment

Q Given a number of regular and irregular objects:
a) Estimate the measure of items
b) Use a personal referent to measure items
c) Measure items using trundle wheels, metre sticks, rulers, tape measures and callipers.
d) Convert the measurements from Imperial to SI system or vice versa as appropriate.

Act Have students list real life examples of how the Imperial system is used. Have the class develop referent units of measurement for the more commonly used units of the Imperial system. For example: $1\ inch$ = diameter of a quarter, $1\ cup$ = a small cup of coffee, $1\ oz$ = weight of a pencil, $1\ ton$ = weight of a small passenger car, $1\ pound$ = block of butter

Q Body Mass Index (BMI) is calculated in kg/m^2 Calculate BMI given for the following situations:
a) Oakley weighs $182\ lbs$ and is $5'10''$ tall.
b) Kelsey weighs $145\ lbs$ and is $1.65\ m$ tall.
c) Marisa weighs $54\ kg$ and is $162\ cm$ tall.
d) Ashtyn weighs $50.5\ kg$ and is $5'4''$ tall.
e) Your own BMI

Q Isaac bought a second-hand treadmill on-line. It would only register in $miles$. Describe a conversion factor which could be used to estimate a conversion from $miles$ to $kilometres$ or vice versa.

Q A jet is flying at the $28\ 000\ ft$. How many $metres$ is this?

Q Estimate each measure:
a) the height of a horse in ft.
b) the diameter of the dial on an IPod in cm.
c) the length of a hockey rink in m.
d) the width of a cell phone in cm.
e) the length of a new pencil in mm.

Q A GPS is set in $miles$. It estimates the distance to a destination to be $188\ mi$.
a) How many $kilometres$ is this?
b) If the speed is registered as $45\ mph$, how many km/h is this?
c) Given this information, what is the estimated time of arrival (ETA) if it is now $10{:}20\ a.m.$
d) Does the answer seem reasonable?

Q A 2×4 stud actually measures $1\frac{1}{2}\ in$ by $3\frac{1}{2}\ in$. To build an interior wall in the basement the stud would be secured to the ceiling and floor and drywall would be nailed on to both of the narrower sides of the stud. How thick is the wall if the drywall measures $\frac{5}{8}\ inch$.

SCO	**M4: Solve problems, using SI and Imperial systems, that involve area measurements of regular, composite and irregular 2-D shapes, including decimal and fractional measurements, and verify the solutions.** [ME, PS, R, V]

[C] Communication **[T]** Technology	**[PS]** Problem Solving **[V]** Visualization	**[CN]** Connections **[R]** Reasoning	**[ME]** Mental Math and Estimation

Scope and Sequence of Outcomes

Grade Nine	Grade Ten	Grade Eleven
SS4 Draw and interpret scale diagrams of 2-D shapes.	**M4:** Solve problems, using SI and Imperial systems, that involve area measurements of regular, composite and irregular 2-D shapes, including decimal and fractional measurements, and verify the solutions.	**PR3**: Demonstrate an understanding of the relationships among scale factors, areas, surface areas and volumes of similar 2-D shapes and 3-D objects. *(FM11).*

ELABORATION

The calculation of area was introduced in Grade 4. By Grade 10 students should understand and be able to apply formulae to find the area of triangles, parallelograms, and circles.

In this course, students will determine an appropriate system and unit of measurement to estimate and calculate the area of various 2-D shapes. For regular polygons, students will be expected to divide the shape into triangles and find the area accordingly.

Students will continue to develop their understanding of both the SI and Imperial systems of measurement and where each is most appropriate, and will be required to solve problems using both systems.

ACHIEVEMENT INDICATORS

- Identify and compare referents for area measurements
- Estimate an area measurement, using a referent.
- Identify a situation where a given unit would be used.
- Estimate the area of a given regular, composite or irregular 2-D shape, using a square grid.
- Solve a contextual problem that involves the area of a regular, a composite or an irregular 2-D shape.
- Write a given area measurement expressed in one unit squared, in another unit squared.
- Solve a problem, using formulas for determining the areas of regular, composite and irregular 2-D shapes, including circles.

SCO **M4: Solve problems, using SI and Imperial systems, that involve area measurements of regular, composite and irregular 2-D shapes, including decimal and fractional measurements, and verify the solutions.**
[ME, PS, R, V]

Suggested Instructional Strategies

- Provide an opportunity for students to draw $1cm^2,\ 1m^2, 1in^2,\ 1ft^2$, and discuss situations where these would be used.
- Students should have the opportunity to solve real life, authentic problems. For example, photos of architecture or quilts would be useful in introducing area problems involving composite shapes. Students may suggest a process that might involve either subdividing the shape into familiar shapes or extending the figure into a quadrilateral and subtracting the missing area. Encourage different strategies. Compare solutions. Pay particular attention to the written form of the solution.

  Subsequent steps would include:
  - taking needed measurements
  - representing symbolically, substituting into formulas, then computing
  - noting appropriate units.
- A Word/Formula Wall would be useful in this section.

Suggested Questions (Q) and Activities (Act) for Instruction and Assessment

Q Estimate how many students could fit on one square meter of the classroom floor if each student is standing and needs about one $12\ inch \times 12\ inch$ tile to stand in. How many students could fit in the entire room if it was empty?

Q One grid chart paper, represent a given composite figure, an irregular figure, or one you created on your own. Illustrate how to calculate the area of the entire figure. Present this to the class.

Q Design a logo for a new app (play park, skateboard park, etc.) incorporating 2D and composite shapes then calculate the area.

Q Imagine you have 50 m of rope.
a) What is the smallest area you can enclose if you use only whole numbers for the dimension?
b) What is the largest area you can enclose with a whole number dimension?

Q You have 50 m of rope and you make a rectangle 1 m x 24 m. How much more rope do you need in order to double the area?

Q Calculate the area of the painted surface in the classroom then calculate how much paint would be needed to do a *Classroom Extreme Makeover*. *(Note to teacher: This could be extended to the entire school.)*

Q A *loonie* ($1 coin) is a regular polygon with eleven sides. It is called a hendecagon. What is the area of the coin if the side length is 7.9 mm and the distance from the centre to the side is 13.3 mm? Express the answer in both mm^2 and cm^2.

Q The dial on an iPod has a radius of ¾ in. What is the area of the dial?

SCO	**M5: Solve problems, using SI and Imperial units, that involve the surface area and volume of 3-D objects, including: right cones, right cylinders, right prisms, right pyramids, and spheres.** [ME, PS, R, V]

[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Math and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Scope and Sequence of Outcomes

Grade Nine	Grade Ten	Grade Eleven
SS2 Determine the surface area of composite 3-D objects to solve problems.	**M5:** Solve problems that involve the surface area and volume of 3-D objects, including: right cones, right cylinders, right prisms, right pyramids, and spheres and using SI and Imperial systems.	**PR3**: Demonstrate an understanding of the relationships among scale factors, areas, surface areas and volumes of similar 2-D shapes and 3-D objects. *(FM11).*

ELABORATION

In Grade 8, students applied formulae to solve problems involving surface area and volume of right prisms and right cylinders.

In this course, this knowledge is extended to cones, pyramids and spheres. For spheres, calculations may include the cubic root with which students may not be familiar. Students will determine an appropriate system and unit of measurement that they will then use to estimate and calculate the surface area and volume of various 3-D shapes. Students will explore relationships between volumes of 3-D shapes with shared dimensions.

Students will continue to develop their understanding of both the SI and Imperial systems of measurement and will be required to solve problems using these two systems.

ACHIEVEMENT INDICATORS

- Sketch a diagram to represent a problem that involves surface area or volume.
- Determine the surface area of a right cone, right cylinder, right prism, right pyramid or sphere, using an object or its labeled diagram.
- Determine the volume of a right cone, right cylinder, right prism, right pyramid or sphere, using an object or its labeled diagram.
- Determine an unknown dimension of a right cone, right cylinder, right prism, right pyramid or sphere, given the object’s surface area or volume and the remaining dimensions.
- Solve a problem that involves surface area or volume, given a diagram of a composite 3-D object.
- Describe the relationship between the volumes of right cones and right cylinders with the same base and height, and right pyramids and right prisms with the same base and height.

SCO **M5: Solve problems, using SI and Imperial units, that involve the surface area and volume of 3-D objects, including: right cones, right cylinders, right prisms, right pyramids, and spheres.**
[ME, PS, R, V]

Suggested Instructional Strategies

- A Word/Formula Wall and models of geometric shapes would be useful in this section.
- A problem solving approach should be used to investigate surface area and volume. An emphasis should be placed on developing an understanding of the derivation of the formulae rather than giving the students the formula and having them apply it. This can be facilitated by using nets or folding geometric shapes.

  Once students understand the formula then they are better able to apply it in contextual problems. Students need to express the answer with the appropriate units.

Suggested Questions (Q) and Activities (Act) for Instruction and Assessment

Q If you double the height of a prism, do you double the surface area? Explain.

Q A cone-shaped pile of road salt is covered with tarps to keep it dry. The radius of the base of the pile is 10.5m and the height is 8.2 m.
a) Calculate the volume of the salt in the pile to the nearest cubic metre.
b) Allowing 15% extra for overlap, what is the area of the tarpaulins, to the nearest square meter?

Q A *Toonie* ($2 coin) has a 28mm diameter. The outer ring is made of a nickel alloy. The inner core has a diameter of 16mm and is made of a copper alloy. The thickness of the coin is 1.8mm. Calculate the volume of nickel alloy in the coin
a) in cubic millimetres
b) in cubic centimetres.

Q The surface area of a softball is about 118cm^2 greater than the surface area of a baseball. The radius of a baseball is 3.7 cm.
a) What is the radius of a softball to the nearest tenth of a centimetre?
b) How many times greater is the volume of a softball than the volume of a baseball to the nearest tenth?

Q A container with a volume of 1000 cm^3 is required to hold a jigsaw puzzle.
a) Find the dimensions for a container with this volume if the container was a cube; a rectangular prism other than a cube; a cylinder; a sphere.
b) Which shape of container would take the lease material to make?
c) Why might your answer in part b) not be the best shape for the container?

Act Use plastic or paper containers to illustrate the relationship between the pyramid or cone and the cylinder. Fill one container with rice and then pour the contents into the next container to illustrates that the pyramid and cone have 1/3 the volume of a cylinder.

SUMMARY OF CURRICULUM OUTCOMES

Geometry, Measurement and Finance 10

[C] Communication, **[PS]** Problem Solving, **[CN]** Connections, **[R]** Reasoning,
[ME] Mental Mathematics and Estimation, **[T]** Technology **[V]** Visualization

Algebra **General Outcome:** Develop algebraic reasoning.

Specific Outcomes

- **A1** Solve problems that require the manipulation and application of formulas related to: perimeter, area, volume, capacity, the Pythagorean theorem, primary trigonometric ratios, income. currency exchange, interest and finance charges. [C, CN, ME, PS, R]

Number **General Outcome:** Develop number sense and critical thinking skills.

Specific Outcomes

- **N1** Solve problems that involve unit pricing and currency exchange, using proportional reasoning. [CN, ME, PS, R]
- **N2** Demonstrate an understanding of income, including: wages, salary, contracts, commission, piecework, and calculating gross pay and net pay. [C, CN, R, T]
- **N3** Demonstrate an understanding of compound interest. [CN, ME, PS, T]
- **N4** Demonstrate an understanding of financial institution services used to access and manage finances. [C, CN, R, T]
- **N5** Demonstrate an understanding of credit options, including: credit cards, and loans. [CN, ME, PS, R]

Geometry **General Outcome:** Develop spatial sense.

Specific Outcomes

- **G1** Analyze puzzles and games that involve spatial reasoning, using problem-solving strategies. [C, CN, PS, R]
- **G2** Demonstrate an understanding of the Pythagorean theorem by: identifying situations that involve right triangles, verifying the formula, applying the formula, solving problems. [C, CN, PS, V]
- **G3** Demonstrate an understanding of primary trigonometric ratios (sine, cosine, tangent) by: applying similarity to right triangles, generalizing patterns from similar right triangles, applying the primary trigonometric ratios, and solving problems. [CN, PS, R, T, V]
- **G4** Solve problems that involve angle relationships between parallel, perpendicular and transversal lines. [C, CN, PS, V]
- **G5** Demonstrate an understanding of angles, including acute, right, obtuse, straight and reflex, by: drawing, replicating and constructing, bisecting, and solving problems. [C, ME, PS, T, V]

Measurement **General Outcome:** Develop spatial sense through direct and indirect measurement.

Specific Outcomes

- **M1** Demonstrate an understanding of the Système International (SI) by describing the relationships of the units for length, area, volume, capacity, mass and temperature. [C, CN, ME, V]
- **M2** Demonstrate an understanding of the Imperial system by: describing the relationships of the units for length, area, volume, capacity, mass and temperature. [C, CN, ME, V]
- **M3** Solve problems, using SI and Imperial units, that involve linear measurement using estimation and measurement strategies. [ME, PS, V, C]
- **M4** Solve problems, using SI and Imperial systems, that involve area measurements of regular, composite and irregular 2-D shapes, including decimal and fractional measurements, and verify the solutions. [ME, PS, R, V]
- **M5** Solve problems, using SI and Imperial units, that involve the surface area and volume of 3-D objects, including right cones, right cylinders, right prisms, right pyramids, and spheres. [ME, PS, R, V

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